OCT 1 1928

Mathematics Teacher

THE OFFICIAL JOURNAL OF
THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

VOLUME XXI OCTOBER, 1928 NUMBER 6

The Evolution of Numbers—An Historical Drama in Two Acts
H. E. SLAUGHT 305

The Influence of General Mathematics on the Subject Matter of Mathematics and on the Theory and Technique of the Teaching of Mathematics - - - - - LAURA BLANZ 318

The Maintenance of Fundamental Arithmetical Skills in the Junior High School - - - - - - - H. E. BUELL 330

Mathematics One Hundred Years Ago - HARRISON C. MESSERVE 336

Address of Welcome - - - - - - - WM. F. SNOW 344

Greetings from the New England Association of Teachers of Mathematics - - - - - - - W. L. VOSBURGH 347

Mathematics and Modern Life - - - - HARRY C. BARKER 350

Is Geometry Possible? - - - - - JEANETTE F. STATHAM 353

The Position of the High School Teacher of Mathematics
G. C. EVANS 357

Published by the
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
LANCASTER, PA. NEW YORK

Entered as second-class matter, March 26, 1927, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized November 17, 1921.

THE MATHEMATICS TEACHER

VOLUME XXI OCTOBER, 1928 NUMBER 6

THE EVOLUTION OF NUMBERS—AN HISTORICAL DRAMA IN TWO ACTS[1]

BY H. E. SLAUGHT

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ACT I: INTERNAL DISSENSION IN THE SECRET SOCIETY OF NUMBERS.

Time: About 1625 A.D. Place: Vestibule of the Secret Chamber of the Society in Paris.

Dramatis Personæ:

The Cardinal: (Representing Whole Numbers, the original Aristocrats.)

The Barbarian: (Representing Negative Numbers, the First Interlopers.)

The Arab: (Representing Zero, the Second Interloper.)

The Hoi Polloi: (Representing Common Fractions.)

The General: (René Descartes in Charge of Personnel.)

Prologue to Act I. (*A young speaker dressed in heraldic costume stands between the parted curtains.*)

My fellow devotees of Mathematics, you will agree that the Number concept is as old as the human race. Let me recall to you some of the struggles through which this concept has come in order to reach its present development. In primitive times there were only a few whole numbers, used for counting, and a few special fractions each with numerator unity. The idea of negative numbers was the outgrowth of the experience of many centuries. It was Descartes in the seventeenth century who conceived the notion of picturing numbers by points on a line extending in both directions from a zero point and thus giving negative numbers and zero a concrete and natural representa-

[1] This playlet may be found helpful in arranging entertainment features for mathematics clubs. It is reprinted here from the AMERICAN MATHEMATICAL MONTHLY by permission of the author and the editor.

tion. The aristocratic whole numbers then became Descartes' positive integers. They had maintained a secret organization, so to speak, for some 3,000 years and had persistently blackballed the negative numbers every time they had sought admission. Likewise zero had been an outcast, a nobody, since the childhood of the race. Even the resourceful Greeks and the mighty Romans were deprived of renown which might have been theirs if they had only been wise enough to admit the humble zero to their clumsy number organizations. What they failed to do the wiser Hindus and Arabs accomplished later on when they brought back to Europe the Hindu-Arabic number symbols and the decimal notation—possibly the greatest invention of all time. Well might Descartes be awarded martial honors for his services as "general in charge of personnel" when he stationed the positive and negative integers and zero as sentinels on his Number Line and then filled in the intervening spaces with the lowly fractions, now recognized as swarming in countless hordes, thus forming the battle front of the mathematical warfare which was to conquer the world.

(*The speaker withdraws from the stage and the curtains open. The Barbarian enters and tries to open the door to the adjoining room, the Secret Chamber of the Society of Numbers. The Cardinal then enters and finds the Barbarian trying to break in. Each actor wears a placard on his breast showing his name in large print.*)

The Cardinal (*Dressed in a cardinal robe*): Is this you, the Barbarian, trying again to gain admittance to the secret chamber of our Number Society?

The Barbarian (*Dressed in a rakish costume*): Yes, General Descartes has told me that we are fully entitled to a place coordinate with you on his new Number Line.

The Cardinal: This is an outrage. We, the Cardinals, have held the major positions in our Society for 3,000 years and it is preposterous now for General Descartes to assign us to positions on the right wing and you to equally important positions on the left.

The Barbarian: But you Cardinals cannot hold the modern battle line alone. It may have been simple enough a few centuries ago when your Number Society was small and there was no need for an extended battle front.

General Descartes (entering in a costume appropriate to the time. See the Open Court Portraits of Famous Mathematicians): The Barbarian is quite right. The number army has been only half organized heretofore and many a battle in mathematics has been lost solely because the army had no left wing. What would you think of an army in the military world that should try to do its fighting with one wing while the enemy was free to execute a flank movement on the unprotected side?

The Barbarian: As an example, there was Captain Cardan a hundred years ago, who needed us Negative Numbers the very worst way in order to complete his victory over the Cubic Equation. We were there ready and anxious to do our part but he spurned our assistance and called us "fictitious" and "absurd."

The General: You Cardinals and Barbarians must quit your quarreling and divide the responsibility of sentinel duty on the Number Line, the Cardinals on the right wing and the Barbarians on the left.

The Arab (Now entering clad in typical Arab costume): Yes, but that leaves an unguarded sentinel post between the wings and I alone, the long despised and rejected Zero, am the only one qualified to guard this point. I am absolutely the center of things, the *King Pin*, so to speak, in this whole army formation.

The Hoi Polloi (Clad in midget costume, entering just as the Arab ceases speaking): But an army must have something more than King Pins and Sentinels! After all, the hordes of Common Fractions are the ones who constitute the real battle front and we are the only ones qualified for this service.

The Cardinal: Well, if we *must* divide the honors with these Barbarians, the Arab and the Hoi Polloi, then our humiliation is complete. We are aristocrats no longer.

The General: But World Progress demands it. You have blocked the world's business for 3,000 years by persistently black-balling these applicants for admission to your Society. Take the Arab, for instance. Without him the ancient Greeks with all their boasted culture could do but little in arithmetic. The mighty Romans could conquer the world in war but they could not master Arithmetic without the help of Zero. It remained for the Hindus and the Arabs to show the world a

decimal number system in which Zero was indeed the "king pin." The Arabs brought this wonderful system to Spain and Leonardo of Pisa introduced it to the rest of Europe; but it took Europe three centuries to fully appreciate this great contribution of the East to the West. Now the Arab and the Barbarian shall come into their own. They shall occupy positions of honor on the Number Line and you Cardinals shall recognize them as your equals. You think it will detract from your glory, but, quite the contrary, it will be your making. You have been sitting around for 3,000 years doing practically nothing but teaching people how to count, while the world's great problems have gone unsolved.

The Cardinal: We recognize our great short-sightedness in this matter and we will now follow your directions.

The General: And another thing! This secrecy business must stop. It began way back in Egypt when Ahmes, the Priest, wrote in the minutes of his famous Papyrus "directions for knowing all dark things." This sort of thing was continued down through the centuries. Someone would solve a problem and carefully conceal the method and then challenge some opponent to find the solution. Even Cardan is said to have gotten his solution of the Cubic from Tartaglia under false pretenses with a promise never to give it away—which promise he deliberately broke.

The Cardinal: Yes, the vow of secrecy was a mistake. We will hereafter do our work in the open.

The General: Finally, you must adopt a Constitution and By-Laws and then examine each applicant for membership on his merits and not act on mere sentiment and prejudice, as you have done all these centuries with the Barbarians and the Arab. The only criterion for membership must be obedience to the By-Laws and allegiance to the Constitution.

The Cardinal: We will do these things, but what about the Arab, the Barbarians, and the Hoi Polloi? Will they also conform?

All in chorus: Yes, we will conform—so say we all of us.

(Two attendants now enter bearing an exhibition of a well-organized Number Line, consisting of a strip of wrapping paper ten or twelve feet long, on which a heavy line is marked off with units and labelled with large figures, positive, negative and zero.

The Cardinal, the Barbarian, Zero, and the Hoi Polloi stand back of this Number Line, while General Descartes stands in front with a pointer indicating the respective subdivisions.)

The General: Henceforth you Cardinals will continue to stand as sentinels on the right wing, and you Barbarians will do duty likewise on the left wing; while the Arab will stand guard at the key position between the two wings, and the Hoi Polloi will fill in the open spaces between the sentinel posts. Thus is the Number Line ready to attack the world's great problems.

(*The Curtain Falls*)

ACT II: INTERNATIONAL ARMISTICE AND PEACE IN THE NUMBER WORLD.

Time: About 1825 A.D. Place: International Open Forum.

Dramatis Personæ:

The Irrational, $\sqrt{2}$: (The Third Interloper.)

The Complex, $\sqrt{-1}$: (The Fourth and Last Interloper.)

The Continuum: (The Solid Front Number Line.)

Professor Dedekind: (Representing the Great Divide.)

Professor Gauss: (Representing the Complex Plane.)

Prologue to Act II. (*Same speaker as in the Prologue to Act I, but now grown old, standing between the parted curtains.*)

My fellow devotees of mathematics, it is now 200 years since General Descartes organized his famous Number Line in which the aristocratic whole numbers were compelled to share the honors of sentinel duty with the negative numbers and zero. During these two centuries a lot of things have happened. When the vow of secrecy was removed from the Number Society as Descartes commanded, there was an immediate renewal of the attempt of the Irrational and the Complex to break into the system. Their struggle for recognition had already been going on for at least 600 years, but encouraged by the success of the Barbarians and the Arab in the seventeenth Century, they then battered harder than ever at the doors of the Number Society till the beginning of the nineteenth century when strong friends like Professors Dedekind and Gauss came to their aid and finally secured their admission. You

are now to hear these actors in the Number Drama speaking for themselves.

(*The herald withdraws and the curtain is opened showing Professors Dedekind and Gauss, in costumes befitting the time* (*see the Open Court Collection of Portraits of Mathematicians*), *walking and talking together.*)

Professor Dedekind: You recall a bold program proposed by General Descartes two hundred years ago when he commanded the Society of Numbers to abolish secrecy and to adopt a Constitution and By-Laws.

Professor Gauss: Yes, I recall the story. It was, indeed, a bold program and it has taken just about two centuries for its accomplishment. Oh, well, that is fast work compared to the slow reforms which General Descartes brought to a climax when he compelled the Cardinals to admit the Barbarians and the Arab to full fellowship in the Society.

Dedekind: It is less than ten years ago since those By-Laws were fully agreed upon and published, namely, the Commutative, Associative and Distributive Laws which really govern our Number System.

Gauss: It took a long time for people to realize that these Laws do actually control the operations of our Algebra and that they are the only criteria for admission to the Society of Numbers.

Dedekind: Of course the Cardinals were the first to adopt these By-Laws, but it took a long time for people to see that the Barbarians, the Hoi Polloi, and Zero all conform to these same Laws, and hence actually belong to the Society just as much as the Cardinals.

Gauss: It does seem awfully slow progress as we look back upon it now, but when we realize how far we have come since Ahmes, the Egyptian Priest, started the organization 3,500 years ago, it seems that we have done wonders in these last two hundred years.

Dedekind: Well, it is up to us to see to it that things move faster in this 19th Century than they did in the preceding centuries. It was a shame and a disgrace to Europe that the Hindu-Arabic symbols and the Decimal notation which were brought to us from Arabia by way of Spain 600 years ago should have

gone almost unnoticed for three centuries before we began to awaken from our long sleep of the Dark Ages to see that Asia had put into our hands what was destined to become the most powerful instrument ever devised by the mind of man.

Gauss: Yes, a great responsibility rests upon us of this generation. Already the members of the Society are getting busy. The Constitution is clear as to our duty. It states that all hands shall add, subtract, multiply, divide, raise to powers and extract roots unceasingly, in season and out of season. That is our job. Thus shall the world's business be transacted.

Dedekind: It is interesting to note that the first and only amendment to the Constitution had to be enacted almost at the very start, namely, that division by zero shall be forever prohibited.

Gauss: Yes, I recall how some well-meaning people have tried to divide by zero and have thought that the quotient was a member of our Society, but have finally been convinced that none of our members can act in that capacity.

Dedekind: There is another curious phenomenon which has caused a lot of trouble in the last two centuries, namely, when our members began to extract roots promiscuously they soon ran across such an anomalous case as the square root of 2 and found that it was neither a Cardinal nor one of the Hoi Polloi. Hence they naturally concluded that such a thing could not belong to our Number Society and so they refused its admission. This case has caused me a lot of trouble and anxiety, but I believe that I now have it satisfactorily explained.

Gauss: That is no more curious than the case that I have been worrying about, namely, the square root of negative one. These two interlopers have caused the Society more trouble than all the orthodox members put together. They have repeatedly applied for admission, and have been consistently rejected for many centuries. Here they come now. Let us hear what they have to say for themselves.

(*The Irrational and the Complex now enter, in costumes appropriate to the age, walking and conversing.*)

The Irrational: I surely have had a rocky time of it. I have tried repeatedly to break into the Number Society but have always failed. They have even told me that I am nobody and have no respectable standing at all. But even Pythagoras

knew better than that 2,300 years ago. He saw that I represent the diagonal of the unit square and that I am just as respectable as the side of the square. Bhaskara in India 700 years ago recognized me as a thoroughly respectable citizen, and the Englishman, Robert Recorde, 300 years ago even wrote a book in which he paid sincere respect to me. But it remained for Professor Dedekind and others of the present time to really vindicate my rights and to atone for the injustice which has been done to me for three thousand years. I want to see him and thank him in person.

The Complex: If you think you have had a rocky time, what do you think of me? I did not have *even one* friend and defendant to do for me what Pythagoras did for you. Not one of the Indian members stood up for me as Bhaskara did for you. I was absolutely an outcast till about 200 years ago when Albert Girard recognized me as a legitimate citizen, while he was solving equations. It is true that Captain Cardan, 100 years earlier, had met me when solving his cubic but he said I was "curious" and "useless." So things went on until a Frenchman by the name of Argand wrote an essay a few years ago in which he defended me and showed why I should be admitted to the Number Society. But nobody paid any serious attention to this essay or to me till Professor Gauss put me on the map and I am hoping to see him and thank him personally for thus atoning for the three thousand years of injustice which I have suffered.

The Irrational (*approaching Professor Dedekind*): Here is Professor Dedekind now. Allow me to thank you, kind sir, for what you have done for me. You cannot know what a relief it is to be admitted to the Number Society after thirty centuries of exile. I know that other friends have been working in my behalf, such as Professors Cantor and Weierstrass, but I am sure your name will go down in history as my emancipator and that the "Dedekind Cut" by which my shackles were broken will forever stand as one of the greatest achievements of mankind. Incidentally you will be pleased to know how gracefully and deferentially the Hoi Polloi have separated themselves into two groups at the "Cut" in order to allow me to take my place on the Number Line. Again, kind sir, let me thank you from the bottom of my heart.

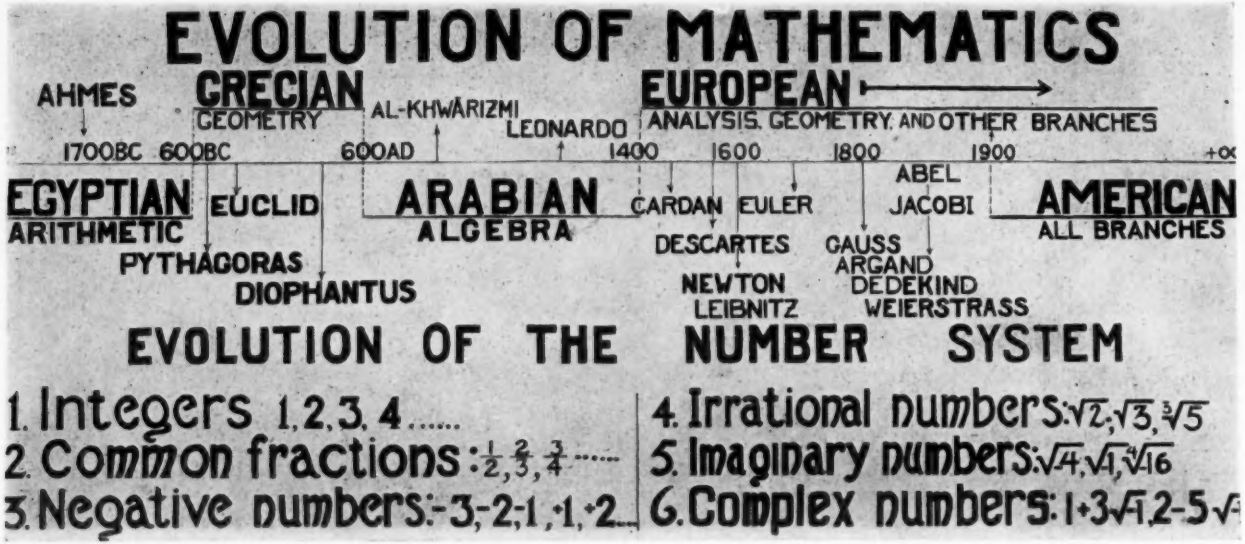
Professor Dedekind: You overwhelm me with your words of praise and gratitude. I assure you that it has been one of the greatest joys of my life to be able to render you this service, especially when I realize that you represent an untold multitude of Irrationals, including the famous π and e, all of whom are now given places on Descartes' Number Line. You have my blessing. Go and take your part in the World's Work.

The Complex (*approaching Professor Gauss*): Ah! Here is Professor Gauss. I am so glad of this opportunity to meet you and to thank you in person for the wonderful way in which you have opened up life's opportunities for me. It was bad enough to be an outcast for three thousand years, but it was the last straw when my friend and companion, the Irrational, was finally given a "place in the sun" on Descartes' Number Line while I was told that there was absolutely no room left there for me.

But just as I was about to die of despair, your brilliant invention, the Gauss Complex Plane, rescued me from oblivion, and now I not only have my place in the sun but I am actually Lord and Master of the whole Number Field, of which the Descartes Number Line is only a small and very special part. I want you to know, kind sir, that I appreciate this great honor and that I am fully conscious of the tremendous responsibility which this wide-open Number Field places upon me. Again, kind sir, let me thank you from the bottom of my heart.

Professor Gauss: You have, indeed, been most patient and long-suffering but your long-delayed recognition has been due not so much to the prejudice of the older members of the Society as to their ignorance. They did not know, and I suppose they could not know, the great possibilities for service that your admission to the Number System will open up. I firmly believe that the 19th Century will justify our highest hopes and will, indeed, see you enthroned as Lord and Master of the whole Number Field.

The Continuum (*A sturdy fellow bearing a placard on which a section of the Number Line is very closely divided to represent the myriads of real numbers*): At last the struggle of mankind to build a Number System adequate to do the world's business is crowned with success. There have been many scenes of con-



This Chart shows the progress of Mathematics, beginning in Egypt, thence to Greece, back to Arabia by way of India, then to Europe by way of Spain, and finally to America—thence to run parallel with Europe for all time.

The dates are given roughly in round numbers. The few names mentioned are chiefly those referred to in the drama.

flict in building this System. There have been jealousies, secret plottings, black-ballings, banishments, but now the mathematical world is at peace. There is complete international understanding and only healthful rivalry. As for myself, I am the symbol of complete union and harmony on the real Number Line. I represent the Solid Front that the Society of Numbers presents to the world. We are now fully prepared to underwrite the world's great problems.

(*The drama closes with all the members of the Company on the stage beneath a banner*[2] *on which is a graphic representation of the Real and Complex numbers. General Descartes and Professors Dedekind and Gauss are standing in front of the Number Group, and all join in singing a stanza of "Hail, hail, the gang's all here."*)

[2] The accompanying half-tone may be helpful in visualizing the historical setting. The original size was seven by three feet. Any club giving the play would do well to make a preliminary study of the historical references. The dates here given are, of course, only roughly approximate in round numbers.

THE INFLUENCE OF GENERAL MATHEMATICS ON THE SUBJECT MATTER OF MATHEMATICS AND ON THE THEORY AND TECHNIQUE OF THE TEACHING OF MATHEMATICS

By LAURA BLANK

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Whether one agrees with the theory of attempting to unify or generalize mathematics or not, whether one entertains an opinion that unification or generalization of mathematics is feasible or desirable even if possible, or one does not, out of this theory and attempt, together with the psychological studies developed contemporaneously, have come new and splendid developments in subject matter, and in the theory and in the technique of the teaching of mathematics. The efforts to unify mathematics have been in a large measure responsible for a general rationalizing and psychologizing of the branches of the subject, not alone the high school branches but the college branches as well. René Descartes, called "the father of modern algebra," the inventor of analytic geometry, perhaps one should say, rather, the discoverer of the interrelations between geometry and algebra, more than three hundred years ago in his complete and unique coordination of these two branches of mathematics set in motion further attempts of the same sort. John Perry, of England, engineer and teacher of engineering students, about three decades ago, dissatisfied with the preparation of the youth of the English schools in mathematics, advocated among other plans for promoting interest, mastery and originality on the part of the students of mathematics that of correlating mathematics with other subjects, with the other sciences particularly, stressing application rather than theory. The most recent impetus in the direction of unifying or generalizing mathematics is the recognition, though no whole-hearted endorsement of the endeavor, made in the Report of the National Committee on the Reorganization of Mathematics in Secondary Education published in 1922.

The very fact that unification, generalization or correlation of

mathematics is so highly debatable a theory or procedure, moreover the fact that the terms "unification," "generalization" or "correlation" of the subject are so vague, indefinite and difficult of definition, the fact that no two proponents of such theory agree as to what is meant by the terms, all of this situation keeps the question open to constant discussion and experimentation. Moreover the growth and spread of the junior high school organization in its various forms, with its inspirational, exploratory and diagnostic courses, general surveys, or previews, more or less superficial, of broad fields—this too has given impetus to the theory of general mathematics.

Hence since the publication of the Report of the National Committee on the Reorganization of Mathematics in Secondary Education, and the subsequent revision of the requirements of the College Entrance Examination Board, this revision being an outcome of the recommendations of the National Committee, numerous textbooks of all sorts have appeared, the great majority of which profess to be general mathematics in graded series for the seventh, eighth and ninth school years. However, there are many books equally interesting and unconventional in algebra and geometry and a few known as general mathematics for the upper high school grades. There are even college books showing the new influence, one of the most unusual and practical of which is *An Introduction to Mathematical Analysis,* by Frank L. Griffin of Reed College, Portland, Oregon, which relates the several branches of elementary college mathematics, trigonometry, college algebra, analytic geometry and calculus. It contains many practical applications, problems collected from scientific, technical and business sources. It is a work possibly best suited to elementary engineering courses, but as a type of text new, suggestive and unique in the college field. Such departures in mathematics are just as welcome in the college field as similar works in the secondary and junior high school fields, since not all students of elementary college mathematics are destined to become research scholars of pure mathematics.

Now, though one might not approve of many of these texts as texts for exclusive use by a class, they are interesting and fertile in suggestions to the student of theory and methods and tendencies in the pedagogy of mathematics. One finds exhibited doctrine as to subject matter both theoretical and in its applica-

tions. In the prefaces and introductions are interesting arguments in support of such doctrine and presentation, particularly if such theory and presentation are distinctly different from those found in the older conventional texts of ten or more years ago.

One finds objectives changing in the teaching of mathematics, however, not alone due to the influence of general mathematics but predominantly as a consequence of the compulsory education laws and the resulting increased school attendance, both compulsory and non-compulsory, and the consequent decreasing fitness of pupils with decrease in intelligence as demonstrated psychologically by such men as Thorndike. Moreover, objectives are changed, due to the influence of the surveys of programs in mathematics of European schools. Smith and Reeve in their recent book, *The Teaching of Junior High School Mathematics*, set up a list of objectives, attitudes, ideals and abilities for cultivation. A study of these objectives shows them to be not specific mathematical abilities alone, but appreciation of the power of mathematics, moreover, objectives psychological in character, recreational, æsthetic, appreciation of the historical growth of mathematics, and the cultivation of certain mental abilities and ideals. Formerly few teachers regarded mathematics as having a marked social value or as contributing particularly to a youth's development as a social creature.

Examination of texts recently produced for junior high school mathematics discloses varied and interesting theories as to the subject matter and its presentation. Many of the junior high school texts are largely arithmetic with a scattering of generalized arithmetic or algebra and some work in mensuration. Another group of texts is predominantly measuring and intuitive geometry, the trigonometry of the right triangle, a small amount of algebra and less arithmetic and little of this of the business arithmetic type. Another group of junior high school texts is for the most part algebra stressing the graph and the formula, intuitive geometry and arithmetic appearing only occasionally. In most of these texts there is a very marked effort to correlate the subjects. The doctrine of one author is to develop only linear functions and linear measuring and linear intuitive geometry in the first stage of the psychological development, then quadratic functions and the measuring of areas and ideas

of surface and area in geometry in the second stage of development of algebraic and geometrical concepts, and finally cubic functions, polynomials and equations, and volume concepts as the third stage in the development, at no point anticipating a later stage in the sequence until the earlier one is thoroughly mastered. Some authors hold that geometrical concepts and space relations are far simpler for the adolescent mind to grasp than algebra with its generalization, and hence geometry should be taught before algebra. These authors cite the historical development of mathematics to bear out their claim. However, there is a very great breach between geometrical concepts or ideas of space relations and a formal geometrical demonstration. We find therefore in the junior high school texts many different theories expressed.

The junior high school texts usually take us through the ninth school year, this last year laying great emphasis upon algebra, due, no doubt, to convention. There are, however, many ninth-grade texts, no part of a series, largely algebra, but treating the trigonometry of the right triangle, graphs exhaustively and intuitive geometry but slightly. The textbooks for the tenth year, undoubtedly because of precedent, are largely or entirely plane geometry, usually demonstrative geometry, though it is in some instances what has been called "anemic geometry." There has been recently a very marked scaling down of the syllabus of theorems for demonstration, postulating or proving informally rather than exhaustively many of the theorems that are more or less self-evident. Even the texts for the tenth grade that are plainly labeled as general mathematics are geometries with inserts here and there of extraneous algebra and trigonometry of the right triangle. Though the trigonometry does have points of contact with some of the theorems, it is so different as to its methods as to seem foreign to demonstrative geometry. Demonstrative geometry is the one secondary school subject which does not correlate with the other mathematical subjects except in its applications. However, it is possible and advisable to use algebraic processes freely to simplify and familiarize the work of geometry, the algebra serving merely as a tool, bringing about nothing like complete correlation. The newer textbooks for the eleventh and twelfth grades do not differ in a marked way from those written heretofore. The subject matter is for the most

part the same, yet the proofs of theory are less rigorous, applications are more numerous and not so far-fetched, and every possible effort is made to coordinate the new subject matter with subject matter already familiar to the pupil. The texts are no longer college books "written down" presumably to the high school level by college men unfamiliar with high school problems.

Not only do the newer textbooks show the influence of the effort to generalize mathematics but instructors are sensing the advantages of coordinating the various branches and of coordinating mathematics with other subjects and interests. There is an interesting personal reaction upon their teaching methods, independent of the textbook used, not alone in junior and senior high school instruction but in the progressive college and university instruction. Possibly there is no branch of mathematics that is totally independent of any other. Each interprets and explains the other and reinforces the other, though it may not ever completely fuse with any other. In every course in mathematics or in any other mathematical science opportunity should be exercised—though not made—of using every possible type of mathematics consistent with the development of the pupils, even at the expense of missing fire occasionally. However, such interrelations should not be made so hurriedly and superficially as to perplex and confuse. Arithmetical processes, already quite familiar to the pupil, form the basis for the development of almost every algebraic operation such as long division, the addition of fractions, multiplication of fractions, reduction of fractions to lowest terms, finding the lowest common denominator, etc., the algebraic process being developed by analogy, step by step paralleling the arithmetical processes. Graphs and formulas can be and should be used universally, both in their interpretation and construction. The pupil should be taught to read, interpret and translate every law of a mathematical nature into a compact formula, if possible. He should be taught the respective advantages of the formula and the graph in numerous and varied specific examples in all of his courses. Moreover, the power of the geometrical diagram can not be overemphasized. This visual space relation makes an added appeal to the purely ideational and worded statement in algebra, in arithmetic, in trigonometry and in many of the other sciences. A diagram convinces and remains when mere generalizations, even particu-

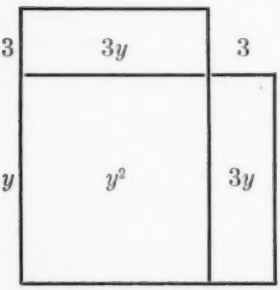
larized statements, fail. In the gradual development of the function concept through a year or several years of mathematics one can draw upon all sorts of mathematical information and ideas. Use can be made of tables of statistics, interpreted by means of economics, of tables of interest, simple and compound, of tables of roots, of trigonometric tables, of graphical explanations and of formulas often involving space concepts. And thus the development continues. What is analytic geometry without algebra, Euclidean geometry and trigonometry? What is the theory of limits and series without algebra, geometrical diagrams, graphs, analytic geometry, calculus? What would the study of theory of equations be without graphs, calculus, geometry? Or differential equations without their applications in physics and psychology?

What in particular are some of the interesting specific correlations? Perhaps the most elementary one is to review the arithmetical processes by bringing them up by means of projects in the field of intuitive geometry or algebra. In the course of such a project, which may involve the finding of the area of the school room floor, for instance, are involved measuring, approximation, estimating the anticipated result, multiplication of arithmetical mixed numbers, "rounding off," and checking the result calculated with the estimate anticipated. Finding the length of the picture molding of the room involves the addition of arithmetical mixed numbers. The student of methods in pedagogy at this point naturally is curious as to whether one's ingenuity is such as to devise projects of such variety and number as to review adequately all of the arithmetical processes which should be reviewed. Furthermore, should such reviews involve drill?

The correlations concerning signed numbers, excluding from our present consideration the operations involving such numbers, are growing more numerous and interesting. Signed numbers may be introduced as designating opposite directions, north and south or east and west, with reference to a fixed point, as north and south latitude or east and west longitude, as distances above and below sea level, as thermometer readings above and below zero, as possessions and debts, as game scores, and as opposing forces, and finally in their use in connection with the daily stock market quotations, showing the number of points rise or fall in a certain stock from day to day.

If fundamental geometrical concepts have been developed previously, the terms of a polynomial when introduced may be associated with the various sides of a polygon so that the entire polynomial or sum of the algebraic terms represents the sum of the segments forming the respective sides of the polygon, that is, the perimeter. From such an introduction the sum of similar terms develops naturally and concretely. Similarly the product of two algebraic factors represents an area, the surface concept having been developed previously. Moreover, building on these same fundamental geometrical concepts, we may revive the old Greek geometrical expansion of $(a+b)^2$ or $(x+2y)^2$, appealing to the eye for a verification of the purely ideational algebraic expansion of a polynomial. Without such devices of visualizing algebra, it, as a subject, becomes arbitrary, dogmatic, of necessity to be accepted on mere authority.

The solution of linear systems and quadratics by the graphic as well as one or several of the purely algebraic methods has been for years accepted practice. However, in connection with the quadratic the expression "complete the square" has long



though perhaps only preliminary to the mechanical algebraic process and therefore an entire area representing y^2+6y+9, a trinomial perfect square. The revived explanation, since lost its significance. Having been given y^2+6y, to complete the figure shown, so as to make a square, we must add a small square so that the side of the added square is 3, its area being 3^2 or 9. We have formed applied possibly only two or three times, at least gives initial meaning to an empty technical term.

Then, for examples of correlation in the field of the trigonometry of the right triangle, the pupil discovers that the trigonometric solution is distinctly less complex than the algebraic-geometrical method using the theorem of Pythagoras, squaring and extracting the square root. Moreover, the instructor will find that simple problems in trigonometry provide excellent applications of approximations in measurement, and computation with measurements, in rounding off results and in the common-

sense handling of statistical tables—all splendid applications of arithmetic.

Considering the subject of ratio and proportion, its correlation with similar triangles and similar polygons constructed to scale provides that visualization so important in learning processes and, moreover, applications and examples of equal importance. The law of the lever, familiarly exemplified in the seesaw of childhood, provides an interesting use of ratio and proportion, a correlation of algebra and mechanics.

In the field of plane geometry considering the common transverse tangent or common direct tangent to two circles, the application of the principle here involved to the pulley provides a correlation with simple elementary mechanics which should interest every pupil. With reference to this same principle concerning tangents, but considering the locus of such tangents, we have another interesting illustration in the explanation of lunar and solar eclipses, a correlation with reference to another science.

In an endeavor to explain such a mathematical concept as infinity, we may approach the subject from various aspects, moreover at a number of different times during the study of different branches of mathematics. Infinity may be regarded as the limiting value of the tangent of an angle as the angle increases from 45° to 90°, thus employing a trigonometric background. We may think of an infinite sum as the limiting value of a series such as $1 + 2 + 3 + 4 + \cdots$ as the series is extended so as to have more terms than can be accounted for by our ordinary system of finite real numbers, thus using, as our background of familiar ideas, number. Or we may consider points in one-to-one correspondence on a line segment and another line of indefinite length, there being for each point on the latter a corresponding point on the former, as illustrated by letting a ray cross both lines. The set of points on the given line segment may be regarded as non-finite, hence infinite in number. This idea of infinity is set into relief by a background of plane geometry.

Considering, in passing, some of the newer worded or concrete problems, there are problems involving mixtures or alloys, involving ratio and proportion and percentage, such as are directly applicable to problems in household arts and chemistry. There are problems concerning loss of weight due to the suspension of

bodies in fluid, such as correlate with the work in physics. There are problems in ratio and proportion such as are used in the planning and cutting involved in sewing, in the enlarging and decreasing of results in cooking recipes, and in similar enlarging and diminishing of plans for application in manual training. These are but a few of the newer problem correlations.

Moreover, since the proponents and textbook writers of general mathematics are so indifferent as to the logical arrangement of subject matter, always maintaining a preference for the psychological, and explaining all rearrangements of subject matter as psychological, in contradistinction to logical, the topics of all of the newer texts are being shifted this way and that, often to very good purpose. In one algebra text, long division is moved to the second half of the book. Though the elementary and fundamental processes of long division, when analyzed and tested, that is, division of monomials, multiplication of monomials, and subtraction, are not difficult as such; yet the entire operation, the process as a whole, is complex and troublesome for some pupils. Again, after a type of special product, examples of which are $(a+b)(a-b)$ and $(l-2)(l+2)$, has been mastered, at once, before taking up a new type of special product, such binomials as a^2-b^2, m^2-9, etc., are factored. This factoring process is mastered before a new sort of special product is approached. Hence multiplication is not completed as a unit before factoring is attempted. Another instance of rearranging the subject matter of algebra is that of teaching the multiplication and division of algebraic fractions before the addition and subtraction of such expressions.

In the field of demonstrative geometry the reorganization of subject matter in some instances is still more radical. The theorems and exercises in some of the newer texts are arranged in units under such headings as congruent figures, parallelograms, similar triangles and polygons, etc., in some cases with a rather daring disregard of Euclid's careful logical arrangement retained for about twenty-three hundred years.

One might continue at length to cite specific examples of the impress of the feeling for general mathematics upon the various branches of mathematics, in its subject matter, and in the theory, method and technique of presenting the various courses. Not all

of the illustrations presented are wholly the result of the endeavor to generalize mathematics. Psychological study of learning and teaching processes in mathematics has contributed its quota to the changes and improvements in the subject. Yet the idea of correlating or coordinating the branches among themselves and with other subjects has done much to humanize and psychologize this important division of learning.

TO ALL MEMBERS OF THE COUNCIL

As stated in the May number of the MATHEMATICS TEACHER we now have practically 5,000 members and we have set ourselves the task of securing 10,000 members by 1930. This means that we must double our present membership. However, this can easily be done if each teacher who is now a member will secure one other. From letters which reach us daily we know that there are still large numbers of teachers who know little or nothing of the work of the Council. We have accordingly published 20,000 circulars advertising the MATHEMATICS TEACHER and the yearbooks, copies of which we shall be glad to send to teachers who will agree to use them in situations where they may do good.

Finally, the Council will shortly issue as a supplement to one of the regular numbers of the MATHEMATICS TEACHER a register of members. It is important that each member of the Council send in at once his name, address, and teaching position as he wishes it to appear in the register. The address blank which was published in the May issue, when properly filled out, should be mailed to the MATHEMATICS TEACHER, 525 West 120th St., New York City. Otherwise the name and address of each member will appear as it occurs on the mailing list unless they are sent in by the subscriber himself.

THE MAINTENANCE OF FUNDAMENTAL ARITHMETICAL SKILLS IN THE JUNIOR HIGH SCHOOL*

By H. E. BENZ

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One underlying assumption of this paper should be stated at the outset in order that the development of the subject may be more readily followed, and in order that those who find themselves unable to agree with this assumption may at once reject any conclusions which may be drawn. That assumption is that arithmetic skills are worth acquiring because of their usefulness in life. Our hypothesis does not necessarily deny the existence of any disciplinary values in arithmetic, nor does it imply a quarrel with those who hold the "rich-present-life-for-the-child" theory of education. It merely insists that one reason for teaching the fundamental skills of arithmetic in the elementary school is that it is the duty of the school to contribute to the future social and civic usefulness and to the happiness, convenience and comfort of the individual by creating in him during the elementary school period the ability to perform accurately and with reasonable dispatch those arithmetical computations which life demands.

It is quite likely that no one here will dispute this hypothesis. I have found no denial of it on the part of those who have written our courses of study and textbooks in arithmetic. Neither is there a general denial of it on the part of those responsible for our arithmetic teaching. In short, we may say that one of the reasons that we teach arithmetic is because the individual will later in life need the abilities and skills acquired.

Let us first turn our attention to some of the evidences of command of computational abilities which junior high school pupils and others display. A recent study[1] carried on in 21 Ohio cities which endeavored to discover the level of ability with

* Read before the Mathematics Section of the Ohio State Educational Conference, April 13, 1928.

[1] Morton, Robert Lee, *Teaching Arithmetic in the Intermediate Grades*, Silver, Burdett and Co., 1927, p. 110.

respect to the fundamental processes with fractions possessed by eighth-grade pupils revealed startling ignorance of these processes. There were 24 examples in each test. Some pupils finished every test with every example correct. However, in the test in addition of fractions, 36 percent scored zero, in subtraction 10 percent scored zero, in multiplication 12 percent scored zero, while in division 13 percent scored zero. The medians for the entire group, together with the fact that there were some who had perfect scores, indicate clearly that the appalling figures just cited are not due to inadequate teaching or lack of teaching of the skills concerned.

In another study Schorling [2] gave tests to beginning seventh-grade pupils covering various items of arithmetical skill and knowledge which are presumably covered in the elementary school. Eighty-one of the 125 items on the test received less than 60 percent correct responses.

Let us examine the results of a similar study [3] dealing with individuals on a somewhat higher scholastic level, made by Touton and Terry at the University of Southern California. They examined the responses given by high school graduates to certain arithmetic items included in the Thorndike Intelligence Examination. The eight items comprising the part of the test in question are taught in the elementary school. But 20 percent of the 504 high school graduates tested were unable to obtain a single correct answer, while only 24 percent got more than two correct. Some got the entire eight correct. When attempts and rights were taken into consideration it was discovered that not one of the items was answered correctly by as many as 60 percent of those attempting it.

Further studies pointing to the same or similar conclusions could be cited, but it is not necessary to pile up the evidence further. Indeed, many will verify the results indicated above by their own experience and observation. It seems then that we may safely say that a large percentage of our high school graduates are being sent out into life, and into our higher

[2] Schorling, Raleigh, *A Tentative List of Objectives in the Teaching of Junior High School Mathematics*, Ann Arbor, Michigan: Geo. Wahr, 1925, p. 24.

[3] Touton, F. C., et al., *Studies of Secondary School Graduates in their Mastery of Certain Fundamental Processes*, University of Southern California, 1927, p. 23.

institutions of learning, without that command of arithmetical computation which they should have. The purpose of this paper is to suggest one way that the existing situation can be improved.

The Psychology of Forgetting

In order that an individual may effectively meet a situation which life presents and which demands a certain skill on his part two things are necessary. One is that the desired skill must at one time have been acquired; the other is that this skill must have been retained during the time intervening between its acquisition and its application. This paper takes the position that at least part of the poor showing made by junior and senior high school pupils, and college students as well for that matter, is due to the fact that the secondary school has made inadequate provision for maintaining those computational skills which the elementary school has labored hard and long to build up. That is, we have taken great pains to teach the elementary school pupil those arithmetic skills which he is likely to need later in life, and have then left him without an adequate maintenance program to insure that the desired skills will be maintained. The inevitable Law of Forgetting immediately gets in its deadly work and it is only a matter of time until the pupil has forgotten what he knew. Attention to first learning is important, but after the first learning, as Gates[4] says, "if the reactions are now given no exercise, disuse gradually results in a weakening of the strength of the connections, which would result in a lessening of the promptness, certainty, and ease of recall, and, finally, in inability to recall."

A few more quotations may help to further clarify the point under discussion. Strayer and Englehart[5] say: "The drill which seems to fix the response to-day will be found insufficient when the same response is called for six months later, unless systematic reviews have been arranged for in the meantime." Strong[6] says: "Reviews at longer and longer intervals are

[4] Gates, A. I., *Psychology for Students of Education*, Macmillan, 1923, p. 258.

[5] Strayer, G. D., and Englehart, N. L., *The Classroom Teacher*, American Book Co., 1920, p. 80.

[6] Strong, E. K., *Introductory Psychology for Teachers*, Warwick and York, 1922, p. 117.

necessary in order to insure that the material will be permanently retained."

The Source of the Difficulty

Here, then, seems to be an important point. We have labored long and faithfully to teach arithmetic properly in the elementary grades, and then have promptly proceeded to nullify our efforts by allowing the skills acquired to deteriorate through disuse. It is my contention that one of the functions of the junior high school is to make a systematic effort to maintain those computational skills which the elementary school has built up. This view is also held by other workers in the field. Thiele [7] suggests "review of fundamental processes" as one of the problems of the junior high school. Hart,[8] writing in the *Fifth Yearbook* of the Department of Superintendence, recommends that more arithmetic be taught in grade 9. The National Committee on Mathematical Requirements, in their report on the Reorganization of Mathematics in Secondary Education, says:[9] "Accuracy and facility in numerical computation are of such vital importance, however, to every individual that effective drill in this subject should be continued throughout the secondary school period." Since we are here interested in the senior high school as well as the junior, it may be remarked that the problem of course persists throughout the senior high school period also. David Eugene Smith, in speaking of the mathematics of grade 10, says:[10] "A single school year serves the purpose and permits also of cumulative reviews of the important parts of junior high school mathematics."

Theoretical considerations as represented by the statements of workers in psychology, as well as tests of the arithmetical ability possessed by junior and senior high school pupils after they have had an opportunity to forget what they learned, indicate a need for more review of fundamental arithmetical skills subsequent to first learning. Let us see what provision is made by textbooks for maintenance of skills.

[7] National Council of Teachers of Mathematics, *Second Yearbook*, 1927, p. 156.

[8] Department of Superintendence, *Fifth Yearbook*, 1927, p. 212.

[9] P. 6.

[10] National Council of Teachers of Mathematics, *Second Yearbook*, 1927, p. 235.

What the Textbooks Contain

It is quite likely that practice in high school mathematics teaching has been influenced more by the report of the National Committee on Mathematical Requirements, above referred to, than by any other single document. As before mentioned, this report includes a sentence which points out the need of continued drill in computation in the secondary school. Curiously enough, however, when the specific recommendations for the senior high school are taken up, maintenance of arithmetical skills is not mentioned as one of the objectives. This illustrates very nicely the attitude that the typical teacher and supervisor is likely to take toward the question under discussion. He is likely to agree to the general statement that it is part of the business of the secondary school to maintain arithmetic ability, and then proceed to lose himself in the new subject matter peculiar to the course under consideration and neglect the matter of maintenance entirely.

Examination of analyses of junior high school textbooks leads to the belief that teachers who follow the textbook will be likely to neglect the matter of maintenance of skill. Most textbooks fail to give evidence of any systematic plan for maintaining fundamental skills, and some indicate that the authors were not even aware of the problem.

In a study of texts made by Hopkins and Paul,[11] reported in the *Fifth Yearbook* of the Department of Superintendence, material intended for maintaining skills is not listed separately. The fact that not enough of this material was found in the textbooks analyzed to suggest to the makers of the analysis a separate classification is in itself significant. In grade 7, work on fundamental processes of arithmetic occupies 8 percent of the space in 13 books. Part of this doubtless is new work, so the amount of space devoted to systematic review is less than the figure quoted. In the eighth grade "arithmetical computation and problems of everyday life" occupy 11 percent of the space in 13 books, while "reviews and tests" occupy 5 percent. Again we find a lack of evidence that the authors of the textbooks considered have a systematic plan for maintaining skills. If they did have, it was not evident enough in any case to impress the individuals making the study under consideration.

[11] Op. cit., p. 202.

In the ninth-grade books the situation is still worse. The only reference to the fundamental processes in arithmetic is in a topic headed "Short cuts in computation," which occupies 0.43 of 1 percent of the total space in 13 texts. Clearly, a teacher blindly following the typical textbook series will be without a systematic plan for maintaining the fundamental skills. A study similar to the one referred to, made by Pieters at Ohio State University, indicates[12] that, of six series of junior high school texts, one had no arithmetic whatever in either the eighth or ninth grade, while only one of the six had any arithmetic in the ninth grade. We are forced to the conclusion that the conscientious teacher who is interested in helping pupils retain the ability to perform the fundamental operations of elementary arithmetic which they have taken the pains to acquire, will not depend on the textbook, but will supply material which will guarantee the desired result. Pieters says that certain questions should be asked in analyzing a junior high school mathematics textbook, and mentions as one of them this question, "What are the educational theories that manifest themselves throughout the work?" In the light of his own analysis it seems that one theory manifest throughout the texts analyzed is the theory that the Psychological Law of Disuse is false.

One solution to the problem arises. Does not the practice which the pupil gets in connection with his other work in mathematics provide for maintenance of the fundamental skills? In answer to this question several considerations come up. In the first place, to assume that it does is equivalent to leaving it to chance, unless we positively know that it does. A scientific point of view of education will not permit us to leave it to chance. Furthermore, the results shown on tests, as indicated in the studies reported earlier in this paper, show that the amount of practice obtained was not enough. But let us see just how much practice is provided. Under the writer's direction the first half of each of four seventh-grade texts were examined. In each case the semester's work was divided into 18 units, corresponding approximately to the work for the 18 weeks. The number of times a pupil would have opportunity

[12] Pieters, C. E., "Some Bases for the Selection of Junior High School Textbooks in Mathematics," *Educational Research Bulletin*, VII, 4; Jan. 11, 1928.

to use any of the four fundamental processes with whole numbers, fractions, or decimals was counted for each week's work. Only a few examples will be given to show that some processes do not get exercise enough to insure retention. In text *A* the pupil will not use either addition or subtraction of fractions after the middle of the semester. In text *B* neither subtraction nor division of whole numbers is used after the eighth week. Only seven examples in subtraction are found. Text *C* provides a thorough review of all phases of elementary school arithmetic during the first three weeks, and then does little more. Only five opportunities to subtract whole numbers occur after that. Text *D* does better, but here we find that multiplication of fractions is dropped after the tenth week of the semester. As might be expected, when we examine ninth-grade textbooks, we find the situation worse. In the first semester of the ninth grade, using book *A*, the pupil will not need subtraction of decimals at all. Text *C* contains a review of arithmetic during the first few weeks. After the fourth week no operation involving fractions or decimals is performed by the pupil. It is not necessary to quote further figures. While the third book in most series puts the emphasis on algebra, our concern is not with how well these textbooks teach what they purport to teach, but how effective they are as tools for the teacher's task, which is to teach new material and to maintain the skills previously taught.

Our examination of the textbooks which teachers use in teaching mathematics in the junior high school leads us to the conclusion, then, that if the teacher is to make teaching truly a fine art, and if she is interested in performing her whole task, which includes that of maintaining skills previously built up, she must have recourse to other material than that included in the ordinary textbook. That this is true is evidenced by the number of supplementary drill devices of one kind and another which are available. Most of these are designed as aids to first teaching, however, and may not prove suitable for the purposes which we have in mind.

A Constructive Proposal

To return to our problem of maintaining skills, doubtless a constructive proposal is in order. When we set about to make such a proposal in the presence of a group like this one, which is likely to be somewhat critical regarding scientific evidence

supporting assertions made, we must proceed cautiously. The situation is indeed somewhat as described by Rugg and Hockett in the following quotation:[13] "Quantitatively the problem of the psychology of practice is reducible to the two questions of how much initial drill is needed to fix the skill, and how many repetitions are necessary to maintain it. No answer at all can be given to the latter question. . . . Granted that we knew how many repetitions were needed at the outset of learning, how often should the practice be repeated? Nobody knows."

Granting, then, that crucial evidence on some points is lacking, may we, at the risk of appearing dogmatic, make a concrete suggestion regarding the maintenance of skill in the junior high school? It is this: It is the duty of the junior high school mathematics teacher to spend some time each week in giving her pupils such drill or practice which shall have for its primary purpose the maintaining of those skills which have been built up in previous grades. In order to be more specific, let us put the amount of time at one half hour each week. I know of no evidence to indicate that that is the correct amount. I am willing to guess that it is better than no time at all. The time will be devoted to carrying through a practice exercise involving the skills to be reviewed. Practice exercises of the type desired are available commercially, or may be constructed by the teacher. Fragment I, on the sheet in your hands, illustrates one type of exercise, which might be used in the seventh grade. Fragment II illustrates a type of exercise more suitable for the ninth grade.

MAINTENANCE EXERCISES

Fragment I

1.	2.	3.	4.	5.
.4728	$37\overline{)5284}$	$4\frac{1}{4} \times 3\frac{1}{2} = ?$	4783	4 hr. 18 min.
$\times .6$			-2856	$\times 7$

6.	7.
Find 27% of $674.35.	A woman bought 6 chairs for $75. What would be the cost of 15 chairs at this price?

[13] Rugg, Harold, and Hockett, John, *Objective Studies in Map Location*, Social Science Monograph No. 1, The Lincoln School of Teachers College, p. 105.

FRAGMENT II

1.	2.	3.	4.	5.
$4ax^2$	Add:	$27\overline{)58236}$	4756	3 3/4
$+3ax^2$	$-5xy$		804	5 5/8
	$7xy$		6894	12 1/2
	$-2xy$		39	6 1/8
			402	

6.

$.06\overline{)42.6}$

7.

$$\frac{a^3b^2c}{ab^2} = ?$$

8.

$$(a-4)(a^2+4a-7) = ?$$

9. Which gives the better rate of income on the investment, a stock paying 7% dividends quoted at 113, or a mortgage that bears interest at 6%?
10. If in a certain triangle the second angle is twice the first, and the third is five degrees larger than the second, find the size of each angle.

One question which arises, of course, is whether such a maintenance program is essential in the other subjects as well as in mathematics. I have no disposition to demand consideration for mathematics which other subjects should not have. Is there not danger, then, that our work will eventuate in a series of maintenance drills, with little or no time left to teach anything new? I think not, but it is possible that a practice exercise similar to Fragment III will meet the objection. It is quite likely that such an exercise, given each week throughout the high school period, would prove of inestimable value in giving the prospective high school graduate a thorough review of the important elements in various subjects just before he completes his work.

FRAGMENT III

1. Spell correctly the misspelled word in the following group:
   counsel comparison nesessity delicious amiable
2. Give the contraction for *we are*.
3. War with Spain was declared by the United States, April 25, 1898, and the treaty of peace ending the war was signed Dec. 12, 1898. How many days did the war last?

4. In a cylinder, $\pi r^2 h$ is the formula for finding what?
5. The Declaration of Independence was signed in what year?
6. Who wrote "The Sketch Book"?
7. Through what country does the lower part of the Nile River flow?

In conclusion, allow me to summarize. The evidence indicates that junior and senior high school pupils do not possess the command of the fundamental processes of arithmetic which we have a right to expect. I have placed the blame for the situation at least in part on our failure to maintain those skills after they have been taught, and have suggested that it is the duty of the junior high school mathematics teacher to give such maintenance exercises as will serve to maintain the arithmetical skills built up in the elementary school. Certain extensions of this proposal were made for the senior high school, and samples of practice exercises were shown to illustrate possible types.

The key-note of this conference has been declared to be "Teaching as a Fine Art." We may think of the art of teaching as bearing a very significant relationship to the science of education. Teaching as a fine art demands that the practice of teaching proceed in the light of scientific evidence available. Little has been said about research in this paper. The intelligent application of certain known facts about the laws of learning seems to point very strongly to the conclusions which I have urged. Our educational research is far enough in advance of our practice to keep us busy trying to keep our practice up to date. Intelligent teaching seems to demand that we do what we can to combat the operation of the law of forgetting, and make some kind of systematic effort in the junior high school to maintain the fundamental arithmetic skills.

If each member of the National Council will get one new subscriber to the MATHEMATICS TEACHER this year, our goal of 10,000 members will be reached by 1930.

MATHEMATICS ONE HUNDRED YEARS AGO*

By HARRISON G. MESERVE

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Although the books I have had access to have been of varying ages, none of them have been exactly one hundred years old. So, strictly speaking, my subject should be "Mathematics about One Hundred Years Ago". The earliest book I have was given to me a few years ago by some friends who found it in the attic of an old house they bought in New Hampshire, and was published in 1797. Other books I have been looking through vary in date of publication from 1822 to 1869.

Incidentally, textbook authors and editors of former years held no mean opinion of their handiwork. The 1797 book is a new edition of Pike's "New and Complete System of Arithmetic", and the editor, Isaiah Thomas, states in his preface: "The numerous Recommendations, by characters celebrated for their knowledge of the Mathematics, prefixed to this and the former Edition, and its introduction as a Classic into several of our Universities, speak its eulogy; and the estimation in which it is now held is superior to any publication of the kind extant." Another interesting item in this preface is the following: "Explanatory of Decimal Calculation in the Federal Currency, the Rules which are adduced under this head, stamp an additional value on this Edition. Dollars and Cents are coming gradually into use; but pounds, shillings and pence will continue to be in practice, and be the basis of many Arithmetical Questions, both in the United States and elsewhere."

In all the books I have examined, the authors, without exception, make much of the new methods and treatment of the subject. Take DeMorgan's "Elements of Arithmetic". DeMorgan was of Trinity College, Cambridge. In his fifth edition (1830, 1832, 1835, 1840, 1846) he says: "At the time when this work was first published, the importance of establishing arithmetic in the young mind upon reason and demonstration,

* Read at a dinner of the New England Association of Teachers of Mathematics, March 24, 1928.

was not admitted by many. The case is now altered: schools exist in which rational arithmetic is taught, and mere rules are made to do no more than their proper duty." And for a text-book written on this side of the water take Colburn and Walton's "First Steps in Numbers" (1848): "It may appear strange to those who have never thought upon the subject, but we believe that children usually pass over many pages of arithmetic, and often go 'through the book', without ever having had any clear idea of the value and use of numbers. The irrelevant answers, or rather wild guesses at answers, which the teacher so often hears, can only be caused by an utter ignorance of the meaning of the words used, and the operation attempted to be performed. The Mathematics are called 'the exact sciences' and arithmetic is made the first step toward a knowledge of them; yet the pupil is commonly taught to test the exactness of an arithmetical operation and the truth of its result, only by reference to the 'Key'. The tendency of such training must not only be to render arithmetic valueless as an exercise of the mind, but, worse than this, to destroy the scholar's confidence in the results of his own reasoning, and to prevent freedom and independence of thought." That reads as though it might almost be from Mr. Barber's "Teaching Junior High School Mathematics". Yet it was printed, you recall, eighty years ago.

Now let us examine some of these old books. Pike's "Arithmetic" (1797) is a book of some 500 odd pages. It starts in with simple addition, subtraction, multiplication, and division, contraction in multiplication, fractions, federal money, method of making taxes, single rule of three direct (*i.e.*, the fourth proportional to three members), fellowship (*i.e.*, partnership), exchange, bills, involution, evolution, arithmetical proportion, geometrical proportion, discount, annuities, miscellaneous questions from physics (falling bodies, pendulum, lever or steelyard, wheel and axle, screw, specific gravity, barometer), various tables, plain geometry (spelled "plain", too), plain trigonometry, measurement of superficies (surfaces) and solids, proportion and tonnage of Noah's Ark, introduction to algebra (carrying one through quadratic equations), and an introduction to conic sections. That book seems to justify its name, "A New and Complete System of Arithmetic". Here are some rather interesting problems I came across:

"How many inches from Newburyport to London, it being 2,700 miles?"

"Suppose your age to be 15 years, 19 days, 21 hours, 37 minutes, 45 seconds, how many seconds are there in it, allowing 365 days and 6 hours to the year?"

And here are two totally illegal questions:

"In 12 pipes of wine how many pints?"

"In 36 puncheons of beer how many butts?"

Suppose we look next at Warren Colburn's "First Lessons in Arithmetic on the Plan of Pestalozzi with some Improvement" (1822). This must be an exceptionally good book, for it bears the following testimonial: "The author has had considerable experience as a teacher, added to a strong interest in the subject, and a thorough knowledge not only of this but of many of the higher branches of Mathematics. This little work is earnestly recommended . . . with the belief that it only requires a fair trial in order to be fully approved and adopted. (signed) J. Farrar, Prof. of Mathematics, Harvard University, 1821." To emphasize his new method, the author states that every combination is commenced with practical problems. As its title suggests, this book is very elementary. The first question on the first page is, "How many thumbs have you on your right hand? How many on your left? How many on both together?" And after a number of variations on that, using different articles, it says, "How many are two and two?" Then it goes on from concrete numbers to abstract numbers. It would be a fairly workable book to-day.

Thomas S. Eaton, of Phillips Academy, Andover, in 1867, brought out a "Common School Arithmetic" for the "large class of pupils whose limited time renders it impossible for them to pursue an extended mathematical course." He takes up the four processes with simple numbers, reduction of compound numbers, English money, Troy weight, avoirdupois weight, cloth measure (2¼ inches make one nail

4 nails make one quarter
4 quarters make one yard),

long measure (3 barleycorns make one inch

12 inches etc.),

chain measure, square measure, fractions, decimal fractions, U. S. money, compound numbers, percentage, and then a miscellaneous section covering ratio, proportion, square and cube root, arithmetic and geometric progression, annuities, permutation, mensuration, and miscellaneous problems.

For an algebra I have W. Colburn's "Introduction to Algebra upon the Inductive Method of Instruction" (1832). He states that his first object has been to make the transition from arithmetic to algebra as gradual as possible. So, "the most simple combinations are given first, then those which are more difficult. The learner is expected to derive most of his knowledge by solving the examples himself; therefore care has been taken to make the explanations as few and as brief as is consistent with giving an idea of what is required. The best mode seems to be to give examples so simple as to require little or no explanation, and let the learner reason for himself, taking care to make them more difficult as he proceeds. This method, besides giving the learner confidence, by making him rely on his own powers, is much more interesting to him, because he seems to himself to be constantly making new discoveries. This mode has also the advantage of exercising the learner in reasoning, instead of making him a listener, while the author reasons before him."

So this book uses the first fifty pages for examples involving all the operations that are ever required in simple equations with one and two unknown quantities. His second example states: "All examples of this kind in algebra admit of proof. As the answers are not given in this work, it will be well for the learner always to prove his results." In his seventh example he has this equation:

$$76 + x = 156 - 3x.$$

"This equation," he states, "is rather more difficult to solve than any of the preceding. In the first place I subtract 76 from both members, so as to remove it from the first member. Then, to get $3x$ out of the second member, which is there subtracted, I add $3x$ to both members; then the x's are all in the first member and the known terms in the other. . . . Thus $5x - 3 = 17$. In this it appears that $5x$ is more than 17 by 3; therefore we say $5x = 17 + 3$. Again $5x = 32 - 3x$. Here it appears that $5x$

is not so much as 32 by $3x$; therefore we say $5x + 3x = 32$. This is called transposition." In other words he develops the theory and use of transposition without calling it by that name until he has finished.

Then he proceeds to generalize his work with equations and explain some of the higher purposes of algebra. He develops his algebraic principles in the more or less standard form: addition, subtraction, multiplication, division, fractions, equations, problems, negative quantities, general formulas, square and cube root, equations of the second degree, fractional and negative exponents, the binomial theorem, arithmetic and geometric progression, logarithms, and compound interest. Throughout all this work word problems are distributed plentifully.

His treatment of negative quantities is worthy of our attention for a moment. "It may sometimes happen as we have just seen, that by some wrong supposition in the conditions of the question, the quantities to be subtracted may become greater than those from which they are to be subtracted, in which case the whole expression taken together, or which is the same thing, the result after subtraction, will be negative. This is what is properly called a negative quantity. . . . It is impossible to add less than nothing, but by a figure of speech we may use the expression, add a quantity less than nothing to signify subtraction. As these negative quantities may frequently occur, it is necessary to find rules for using them." And so he develops his rules.

Again, regarding division by zero, he shows how, when the divisor is actually 0, the quotient must be infinite. "From this consideration, mathematicians have called the expression $a/0$ a symbol of infinity. But as infinity is an impossible quantity, I prefer the term impossible, as being a term more easily comprehended. I shall therefore call $a/0$ a symbol of impossibility."

His treatment of the binomial theorem, he is careful to state, is entirely original. It makes interesting reading to one who has previously studied the subject, but I am afraid that it would be hopelessly confusing to one who was studying it for the first time.

For a geometry I have Thomas Hill's "First Lessons in Geometry" (1855) and Bernhard Marks's "First Lessons in Geometry" (1869). Both of these books are designed for young children. Thomas Hill was President of Harvard from 1862

to 1868. He asserted that children of seven were quicker at comprehending first lessons in geometry than those of fifteen. "I have addressed the child's imagination, rather than his reason, because I wished to teach him to conceive of forms. The child's powers of sensation are developed before his powers of conception, and these before his reasoning powers. This is, therefore, the true order of education; and a powerful logical drill, like Colburn's admirable first lessons of arithmetic, is sadly out of place in the hands of a child whose powers of observation and conception have, as yet, received no training whatever." Marks, in his "Geometry", states: "The prime object of school instruction is to place in the hands of the pupil the means of continuing his studies without aid after he leaves school. . . . Children are taught to read, not for the sake of what is contained in their reader, but that they may be able to read all through life, and thereby fulfil one of the requirements of civilized society. So, enough of each branch of science should be taught to enable the pupil to pursue it after leaving school. If this view is correct, it is wrong to allow a pupil to reach the age of fourteen years without knowing even the alphabet of Geometry. He should be taught at least how to read it." And so Marks presents in this book a more or less typical, and shall we say modern, course in observational geometry. His use of color in his diagrams is rather strikingly interesting.

President Hill in his book carries his geometry much further along through a very interesting discussion of curvature, taking up cycloids (about a wheel rolling), epicycloids (wheels rolling round a wheel), where we come across one called "the cow's foot in a cup of milk", hypocycloids (of a wheel rolling on the inside of a hoop), catenaries (about a hanging chain), conic sections through a study of shadows and pieces of string, and one chapter on solid geometry. When he begins his work with cycloids he says: "You will have to study Geometry a good while, before you can prove the other interesting things which I am going to tell you. You can easily understand what I am going to tell you; but you cannot understand how I know it." It really is a fascinating little book.

After this brief survey of these older books, the question naturally arises, "Wherein do they differ from our textbooks of to-day?"

The most striking difference is in the manner of their approach. As a "carryover" from the old days when all knowledge was supposed to be in books and going to college consisted in "sitting in" at lectures where you attempted to take down the lectures verbatim so that you could have accessible the knowledge the professor had, these textbooks did not presuppose prior knowledge of the student. Marks's "Geometry" states: "This little book is constructed with reference to being used by teachers who have themselves no knowledge of Geometry." Consequently the old books are written with more detailed explanation than present-day books.

As regards their treatment of arithmetic, the old books unquestionably cover a much more extensive field than we care to cover to-day. This is doubtless due to the importance of arithmetic in those days, this importance being caused partly by the inherent importance of the subject itself and also by the extremely restricted field of knowledge deemed worthy of study. Our present emphasis upon rounded numbers was ignored, as is shown by the answer "75 and 2451/20720 days" for the time necessary for a certain number of men to dig a ditch. (DeMorgan's "Arithmetic", 1846, page 150.)

But in at least three respects those old arithmetics are not so very different from our present-day ones. First, the so-called reverse method of multiplication is found in some of those books, namely, Eaton's "Common School Arithmetic", page 39 (1867), and also multiplication to a certain number of significant figures in DeMorgan's "Arithmetic", pages 84 and 87 (1846). Secondly, the insistence upon checking or proving the answer by the pupil himself is right in line with present-day practice. And third, our present courses in general mathematics are distinctly analogous to the unified mathematics the old books talked about. All the old arithmetics include some algebra and geometry and even some trigonometry. A striking similarity to to-day's textbooks is shown by the care each author takes in stressing the fact that his is a new book, in line with the new methods of the day, emphasizing the practical use of mathematics. "The supplements to the rules and the geometrical demonstrations of the extraction of the square and cube roots, are the only traits of the old work preserved in the new." (Adams, "Arithmetic", 1829.) W. Colburn, in his "First

Lessons in Arithmetic" (1822), says: "In this edition every combination is commenced with practical examples." Furthermore, the authors all protest against the earlier methods of proof, which consisted in the use of a Key, emphasizing the need of checking: "Why should he not be from the first thrown on his own resources and held strictly responsible for the accuracy of his work?" (Colburn, "Arithmetic", 1855.)

Colburn's "Algebra" (1832), with a very few modifications, could be used as a new book to-day. The main omission we should have to make would be the section on cube root, a substitution for his treatment of the binomial theorem, and, in addition, a new chapter for the treatment of graphs and the trigonometry of the right triangle. Just as in the old books the emphasis was upon practical use, even though it carried over into odd fields at times, so, too, this algebra has for its core the problem, to which everything else is subordinated. The over-refinements of purely manipulative processes, so manifest in the textbooks of twenty-five years ago, are marked by their absence in this book.

As regards observational geometry I think I have said enough to show that our modern or new geometries are not very different from the old.

My reaction to all the books I have looked through is that the new mathematics is not so very new after all. Most of what we call the new can be found in the old books. Where the new does seem to differ radically from the old, a closer inspection seems to justify the conclusion that the new has been applying common sense to its practices. I hope I am not going to be misunderstood. I do not ask for the books of yesteryears for my classes. I prefer most of the new books. But basically, because we are dealing with eternal verities, there is nothing new in what we are teaching. We are going at it from a slightly different viewpoint. Our compass still points the same way it did fifty and a hundred years ago.

ADDRESS OF WELCOME [1]

By WM. B. SNOW

Assistant Superintendent of Schools, Boston, Mass.

I feel greatly honored at being designated to extend to you this evening the cordial welcome of the School Department of the City of Boston, and I confess to an especial pleasure in being privileged to extend that welcome to the Mathematicians rather than to the Scientists or the Historians or the Classicists or the specialists in English. For although most of my friends in educational circles now think of me as a mere dabbler in the modern languages, I might assert, in the words of Spartacus to the gladiators, that "I was not always thus," for in the early days of my teaching I foregathered with men who talked of cosines, integrals, quaternions, and transcendental functions; and though our paths long since diverged, I can imagine, in meeting you tonight, something of what one feels when, after many years of happy wedlock with another, he meets again with her who charmed his youth. The names, too, on your program: Mr. Vosburgh and Miss Kee, of our Boston Teachers College; Tyler, of the Institute, whom I first knew when we were boys together in a little New England town; Downey, who taught for years beside me and succeeded me as Headmaster of the English High School; Barber, who has worked with me in both day school and evening school and whose powers developed in that same English High School in contact with such colleagues as Evans and Wright, and Marsh! Why, the very atmosphere of that school was redolent of mathematics; two thirds of its headmasters were noted mathematicians, and the line runs straight from Emerson and Miles through Thomas Sherwin and Edwin P. Seaver to Walter F. Downey, who is to speak to you tomorrow afternoon. How could I fail to rejoice in finding myself in such company! So, in calling you friends, I almost venture to add "fellow-mathematicians," for may not Echippus claim kinship with Bucephalus?

[1] Delivered before The National Council of Teachers of Mathematics at the Boston Meeting, February 24, 1928.

Probably the most widely known of Boston's mathematicians was Nathaniel Bowditch, whose *Practical Navigator* was issued in 1800. Bowditch was actuary of the Massachusetts Hospital Life Insurance Company; President of the American Academy of Arts and Sciences; skilled in physics and astronomy; and famous for his translation of Laplace's *Mécanique Céleste*. His library is one of the most important collections now contained in the Boston Public Library.

My function is not, however, to enumerate the distinguished mathematicians who have lived in and about Boston, but rather to bid you welcome to all that our school department can offer you, to express our pleasure in having you here, and to hope that your visit may be in every way agreeable and profitable. Speaking for myself alone, may I confide to you certain misgivings as to whether we are in every respect maintaining the traditions of our past, and bespeak your aid, if these misgivings have a substantial basis, in restoring our former excellence. This little book is the *Boston Almanac* for 1849, which contains a detailed description of the public schools of that day. I find in it this statement: "The present school system of Boston is nearly complete and almost perfect." I wonder whether Mr. Vosburgh and Mr. Downey would dare today to make as strong a statement.

This other little book is entitled *First Lessons in Geometry*, by Thomas Hill, once President of Harvard College. It was copyrighted in 1855, a little later than that happy year when Boston's school system was "nearly complete and almost perfect." It will give you an idea of the mathematical proficiency of Boston school children in 1855. Its motto, printed on the cover and on the title page, is "Facts before Reasoning." So don't let the present-day apostles of observational geometry flatter themselves that they are pioneers! The author says, in his preface, "I have long been seeking a Geometry for beginners suited to my conviction of what is a proper foundation for scientific education. Meanwhile my own children were in most urgent need of a textbook, and the sense of their want has driven me to take the time necessary for writing these pages. Two children, one of five, the other of seven and a half, were before my mind's eye all the time of my writing."

Having presented straight lines and curves, triangles, quadrangles, and circles, similarity and isoperimetry, he proceeds to

discuss radius of curvature, evolutes, cycloids, epicycloids, and hypocycloids, catenaries, the conic sections, and caustic curves, following these by a few pages on the sphere and some practical questions and problems for review.

Now in my recent visits to Boston schools, I have not only failed to find the average child from five to seven and a half years of age well-informed on all these topics, but I have been forced to the conclusion that not all of our high school pupils could pass successfully an examination on catenaries and epicycloids. This is the cause of my misgivings; and if as a consequence of this meeting, the Boston schools can be restored to their earlier state of approximate perfection, and if your discussions here result in a mathematical renaissance such that Bostonians from five to seven and a half years old will again revel in caustics, evolutes, and hypocycloids, we shall long remember your words with joy and think upon your presence with thanksgiving.

THE NATIONAL COUNCIL YEARBOOKS

The attention of the members of the National Council is again called to the importance of securing copies of each of the yearbooks before it is too late. Mr. Austin reports that his supply of the First Yearbook is now exhausted. The Second Yearbook on "Curriculum Problems in Teaching Mathematics" may still be secured from the Bureau of Publications, Teachers College, 525 West 120th Street, New York City, for $1.25 (Bound Volume $1.50). The Third Yearbook on "Selected Topics in Teaching Mathematics" may also be secured from the Bureau of Publications for $1.75 (Bound Volume).

GREETINGS FROM THE NEW ENGLAND ASSOCIATION OF TEACHERS OF MATHEMATICS[1]

By W. L. VOSBURGH

President of the New England Association, Boston, Mass.

In extending greetings to the National Council of Teachers of Mathematics from the Association of Teachers of Mathematics in New England, it seems fitting and proper to mention a few facts regarding the origin and development of the Association which, as its president, I represent and which Association is about to celebrate its 25th Anniversary.

A complete summary of "Twenty Years of the Association of Teachers of Mathematics in New England," as compiled by a committee of which Professor H. W. Tyler, of Massachusetts Institute of Technology, was chairman, is given in the MATHEMATICS TEACHER for February, 1924. I am indebted to this summary for the facts which I shall give.

Quoting from this summary, "The need of some organization which should bring together college and school teachers of mathematics seems to have been recognized by various persons independently interested about 1900." (End of quotation.)

The names mentioned in this connection are: Professor Osgood, of Harvard; Professor Tyler, of Technology; and Messrs. Evans, Campbell, Nichols, Francis, Curtis and Hagar, representing both public and private secondary schools.

On April 3, 1903, a printed letter was circulated in which it was proposed to form an association of the New England teachers of mathematics and the announced date of the first meeting was April 18, 1903, and the place the building of the Boston Latin School. In this letter the following objects of the proposed association are given:

I quote,

"To give to elementary mathematics a more important place in the training and thought of the pupil, and to make the instruction in mathematics more effective and more closely re-

[1] Delivered to The National Council of Teachers of Mathematics at the Boston meeting, February 24, 1928.

lated to practical affairs; to increase the teacher's interest in the science of mathematics, and thus to react beneficially on his teaching."

On the Program of this first meeting (April 18, 1903) there was a paper on "Observational Geometry," by Mr. Campbell, while on the Program of the second meeting (November, 1903) there was a paper on "Graphical Representations in Arithmetic and Algebra," by Mr. Evans. In view of the prominence given to Intuitive Geometry and Graphs in our modern courses of mathematics, it is interesting to note that their importance was anticipated twenty-five years ago.

Throughout the twenty-five years of its existence, this Association has been able, fortunately, to maintain and preserve a happy and helpful balance of its two forces, college Professors of mathematics and teachers of secondary school mathematics, so that, as Professor Tyler states: "In the meetings and social gatherings of the association no boundary exists between school and college teachers, and in discussion there is free and profitable give and take." This spirit of cooperation between college and secondary teachers probably accounts, as does nothing else, for the growth of this Association from a membership of 170 in 1904 to a membership of 520 in 1928, with its two branches, Connecticut Valley Association and Providence Association.

This balance of forces is maintained in its organization: when a school man is chosen for president, a college man is chosen for vice-president, or the reverse has always been true. On its Council of six members, outside of its officers, two are always college representatives. In its programs, while there are always papers dealing with the teaching of certain topics in secondary school mathematics intended to be helpful to inexperienced teachers, there is always at least one paper dealing with some advanced topic in mathematics by a college teacher. This Association is indeed very much indebted to the great number of college teachers of mathematics in the institutions throughout New England for the help and inspiration which they have given to it by appearing from time to time on its programs. In addition to the inspiration given by these college teachers, the Association has been able to attract to its programs college and secondary school teachers from other sections of the United States and even from England.

Through its publications from time to time (a list too long to mention here), about every phase of mathematics has been studied and reported upon by committees of this Association, so that Professor Tyler, in his closing paragraph of his summary, felt that he could justly say: "In the recent far-reaching work of the National Committee, college and school men and women have cooperated on equal terms. It is not too much to say that for this fortunate change in mathematical education this association deserves a substantial share of the credit."

This Association, with its record of twenty-five years of constructive work in attempting to improve the teaching of mathematics, may therefore appropriately welcome an organization such as the National Council of Teachers of Mathematics, whose announced object is to assist in promoting the interests of mathematics in America, especially in the elementary and secondary fields.

If this National Council can reach teachers of mathematics through its meetings and published reports in all parts of this great country of ours as our association has been able to reach teachers throughout New England, great good will be done for the cause of mathematics and this association will be glad to cooperate in so worthy a cause.

"The effort of the economist is to *see*, to picture the inter-play of economic elements. The more clearly cut these elements appear in his vision, the better; the more elements he can grasp and hold in his mind at once, the better. The economic world is a misty region. The first explorers used unaided vision. Mathematics is the lantern by which what before was dimly visible now looms up in firm, bold outlines. The old phantasmagoria disappear. We see better. We also see further."—PROFESSOR IRVING FISHER.

MATHEMATICS AND MODERN LIFE[1]

The Challenge and the Opportunity

By HARRY C. BARBER

The Phillips Exeter Academy, Exeter, N. H.

Present day demands that each subject justify its existence in the school curriculum are so well known as to need no review in this paper. The bearing of these demands on mathematics, however, presents an interesting situation when considered in relation to the increasing importance of mathematics in modern civilization. It is a curious phenomenon that mathematics should be challenged just at this time when its value in the world is greater and more apparent than ever before. This challenge does not so much question the value of mathematics itself as the objectives of mathematics teaching, the content of mathematics courses, and the methods of presentation. The mathematics of the secondary school may well be criticized as being too mechanical, too much concerned with technique, too little concerned with the true kernel of the subject. The leaders within the field of mathematics teaching are in hearty accord with these criticisms, and they are making a constant battle against (1) too meagre a definition of mathematics and too narrow a concept of its possibilities, (2) the use of obsolescent material, (3) the rote method of presentation.

It is interesting to note that the reorganization in mathematics taking place in this country is paralleled by similar reorganization abroad. The new objectives emphasize the understanding of concepts and the appreciation of the power of symbolism. They stress the use of discovery and independent thinking. Algebraic technique becomes a tool to be mastered for its use as a means to an end: it is not an end in itself. In algebra, the emphasis is being placed on the formula and the equation, on the study of the relations between numbers, and on problem solving. Other topics are losing in importance.

[1] Address before the National Council meeting in Boston in February, 1928.

Present day aims in algebra, for example, avoid the danger of making its development parallel to that of arithmetic. Accordingly, the newer treatment of algebra pays less heed to the "four fundamental processes" than did the one which it is replacing. Factoring is reduced to three or four cases. The work on fractions and parentheses is greatly diminished and radicals and exponents are approached from a different angle. In the time thus made available for other work the pupil now studies numerical trigonometry, devoting considerable attention to practical computation. The guiding principles in making these additions and omissions are like those which govern modern curriculum revision in general: to be retained, a topic must have value intrinsic to itself, it must have educational value, and it must be within the grasp of the pupil.

An essential and vital part of the reorganization of elementary algebra comes in its method of presentation. The traditional treatment of algebra tended to emphasize technique rather than comprehension. The newer treatment has the aim of making the work thought-provoking and more truly educative. It must not be inferred that all teachers of the past generation erred in making algebra a mechanical exercise nor can it be said that all teachers of the newly organized course are successful in subordinating the manipulative aspects of the subject. The point must be stressed, however, that the treatment of the subject matter is a vital and essential part of the reorganization.

Let us consider the study of signed numbers as an example. In view of the length of the period in which the concept of the negative number developed historically, it is not surprising that the beginning student cannot master it at once. The introduction of the negative number, necessitating as it does the reorganization of our rules for operations, must be approached gradually and with great care lest the student lose his comprehension and turn to the mechanical following of rules. One valuable method of introducing this subject is through the familiar notion of a *shortage* which, it may be added, is also one of the ideas through which the human race reached its first concept of a negative number. This use of the shortage idea permits the student to develop his rules of signs for himself and it allows him to become familiar with the general principles of negative numbers before he is confronted with their more formal aspects.

Another innovation which should be mentioned is the discouraging of the use of certain terms which are thoughtlessly applied by the pupil who frequently cannot tell their precise meaning. Chief among these are the terms "cancel" and "transpose." The first of these has lost its significance through its blanket use for the processes of addition of numbers with like absolute values but with unlike signs, for the division of numerator and denominator of a fraction by a common factor, for the subtraction of like numbers from both members of an equation, and the like. "Transpose," similarly, has become the name for a mechanical operation effected without regard for the process actually used. In each case, work is more thought-provoking when the name of the process actually used is substituted for the catch word. It is more than likely that errors in computation are avoided by directing the student's mind to the process by the use of its specific name, rather than by slurring over the process by the use of a somewhat meaningless term.

It does not follow that this newer organization of algebra is diluting the subject nor that it makes for too easy a course. The subordination of mechanical drill and the stressing of thought exercises need not necessarily have this effect. This consideration, however, makes one realize how close the cooperation must be between the leaders in this work and the other class room teachers so that our treatment of the several topics may capitalize their innate possibilities. Class room experiment must play a great part in this work and the critical judgment of expert teachers must be utilized.

The challenge before secondary school mathematics, then, is to justify its presence in the curriculum by its value in the life of each pupil. Its opportunity is to review and reorganize its offerings, adapting its content and its method to the end that each student may be enabled to understand the value of mathematics in making and interpreting the world about him and to the further end that each may have experienced the type of thinking for which mathematics stands.

IS GEOMETRY POSSIBLE?

By JEANETTE F. STATHAM,

St. Agnes School, Albany, N. Y.

The attitude of many teachers toward geometry, as shown by frequent discussions of this subject, might make some students even more reluctant to take it up than they are now. Students pass along the word that geometry is hard and most sophomores apparently try to avoid one of the most beneficial subjects in the high school curriculum. If they could approach it with unprejudiced minds, avoiding the deadly "complex" which many of them talk about with delight, the teacher's task would be much easier.

One can never expect to have a class of brilliant students, every one of whom is a mathematical genius. It would be far easier to find a group without one student who had a genuine fondness for mathematics. Our task, then, is to inspire the ordinary group with a desire to get all they can out of the study of geometry, not only in order to pass the subject at the end of the year, but also because of the many incidental benefits.

There are numerous ways in which the recitation can be varied, so that monotony may not lower the vitality of a class mentally, but always concentration is the secret of success. Many—perhaps most—students reach the second year of high school without having learned to concentrate even fairly well. When they are required to do something that demands the closest attention, they soon weary and we must change often to work of a different kind, in order to lessen the mental tension. Gradually concentration becomes a habit.

The amount of work required in preparation for C.E.E.B. or Regents examinations is undoubtedly more than ordinary students can master during a school year, partly because they have not learned to reason or to concentrate before beginning the year's work. They could easily grasp the meaning of the theorems given and probably learn to prove them, but to apply the principles learned to original problems is another matter.

Simple numerical problems are easily worked, but the ability to prove more or less difficult original theorems, or to work complicated numerical problems is not acquired by any but very good students in the ordinary class work of one school year.

Variety helps to hold the interest, but there must not be so much of it that confusion results. Variety can be systematized until it virtually becomes routine and the numerous combinations of geometric principles can be constantly repeated until they become part of the student's subconscious knowledge. As the MATHEMATICS TEACHER has set forth many helpful ideas in the past months, may we be pardoned for passing on some suggestions that have brought results in plane geometry, most of which can be adapted to other courses equally well.

As soon as the student understands the meaning of vertical angles and why they are equal, he unconsciously lets his mind wander when that subject is under discussion. If, however, he is required to listen to an oral proof, for which he must carry the figure and letters in his mind, he cannot think of other things or he may be questioned about some detail he is not following. To cultivate the imagination, then, is one of the first aims of the teacher of plane geometry. A student who learns to visualize his mathematics and to do mental algebra and mental geometry as easily as he should learn to do mental arithmetic is far on the road towards his goal.

After a theorem has been well explained and possible variations in shape of figures have been illustrated, power in concentration can be gained rapidly by requiring oral proof. Of course, the more complicated proofs are not suitable for such treatment, but it is surprising how even dull pupils will wake up and participate in this part of the recitation. After full oral proof has been given one day, it is often necessary to ask only upon what the proof of a certain theorem depends and a large proportion of the class will probably recall the mental picture and reply correctly. The letters should generally be different from those in the book, to guard against memorizing the proof.

A good way to begin a recitation is to give a test question based on the lesson for the day and to have this handed in with the written work. Pupils who expect this searching test do not merely write out their lesson and forget what they have learned,

but they study for future use. As not over five minutes can usually be spared for this part of the program, the questions must be carefully formulated by the teacher. Numerical problems are of great value.

In addition to the test question, or in combination with it, dictation is invaluable in developing concentration. Students are told that they must execute what is given exactly as it is dictated, as nothing will be repeated. There is unlimited material from which to select. A new theorem, which is to be a part of the next day's lesson, may be dictated, with directions as to what is given. What is required should be omitted. Successive steps in the proof are dictated, for which the pupils write their own reasons. At the conclusion of the dictation the students are asked to state clearly what has been proved. Such choice of material does not waste time, as the new lesson has been partially worked out and can be assigned without further explanation.

To encourage work on original theorems, for which there is not time in class, special propositions may be assigned for outside work, with the understanding that they are to receive extra credit if done correctly by the student, with only such help as the teacher may furnish. This stimulates the students who are preparing for college and must have special training for their dreaded examinations, but it does not lower the grade of those who do not do them, as it is not required work. The statement of important theorems worked out in this way is required as a basis for future work. To dictate a "special" some time in advance of its assignment makes it easier of solution when it appears.

Most profitable of the plans used is the test each Friday on the week's work. No new assignment is made for that day, so that the students may have time to review thoroughly the four days' work. The "specials" for the week must be in on Friday, if they are to receive credit, so a little extra time can be used to advantage. By reviewing each week's work as we proceed, we more than compensate for any loss that may come from the slightly longer daily assignments. A teacher can estimate more accurately what the individual students are really getting out of the course by the weekly test than by any other means.

These methods may seem to require more time than can be

spared during the school year, which is much too short for so comprehensive a subject as plane geometry. But it will soon be evident that the increased ability resulting from such work enables the student to proceed more rapidly. It is quite possible to complete the required course by the first part of April. This leaves the rest of the year for thorough review and for more difficult applications of the entire group of propositions studied. At least two hundred so-called "originals" can easily be worked during the weeks devoted to review.

EDITORIAL CHANGE

Dr. J. R. Clark has resigned his position at New York University to accept the principalship of the High School at the Lincoln School of Teachers College, Columbia University. Dr. Clark's new duties make it impossible for him to continue as one of the editors of the MATHEMATICS TEACHER and the National Council Yearbooks. The National Council is glad to announce, however, that Dr. Clark will remain on the editorial committee of the National Council.

Beginning with this issue, Professor W. D. Reeve of Teachers College, Columbia University will assume the duties of editor and business manager of the MATHEMATICS TEACHER and the National Council Yearbooks. All communications relating to either of the above publications should be addressed either to Professor Reeve or to the MATHEMATICS TEACHER, 525 West 120th Street, New York City—except orders for the yearbooks which should be addressed to the Bureau of Publications, Teachers College, 525 West 120th Street, New York City.

THE POSITION OF THE HIGH SCHOOL TEACHER OF MATHEMATICS[1]

By GRIFFITH C. EVANS

Rice Institute, Houston, Texas

There are many things that make the present position of the high school teacher unsatisfactory and of doubtful portent for the future. A particular misfortune dogs the teacher of mathematics. Perhaps only a little can be done about it, but we may at least look the situation in the face. Moreover, even if merely small things may be done at present, an awakened public conscience and love of parents for their children may eventually furnish the lever which will change a small movement into a considerable reform.

Is there any doubt that the high school teacher's position is unsatisfactory in general, and that of the mathematics teacher in particular? In some ways the situation in the primary and grade schools is preferable; the teacher is regarded as a substitute mother and has a warm place in the affections of parent and pupil, and she herself, generally young, keeps the little children in affectionate regard as temporary substitutes for her own. But in the high school the relation is different, and the contact with parent and pupil is not so close. The teacher is an insignificant part of the process, a cog in the machine, a link perhaps between the superintendent and the magnificent school buildings, but much less important than either.

In regard to mathematics in particular, we know one young man who had made somewhat of a success in his teaching of this subject. His principal said he had done very well, that he would promote him, and thereafter he would teach English! It is as if anyone could teach mathematics, and as if all you had to do were to get through so many pages a day by the project method or some other method, and look up the answers in the key. There are a few tricks to learn, like getting fractions over a common denominator whenever you see them, or how to solve

[1] An address delivered at the meeting of the Mathematical Association at Duke University, Durham, N. C., April 14, 1928.

a quadratic for x, transposing this and that; but these things are covered by a few rules. There is an emphasis on tests of drill as if the important thing were that the student should be able to do so many operations per minute! I suppose the public does not realize that it is absurd to teach mathematics as if it were typewriting, or that there is a difference between training and education. The public is not to be blamed—but there are people who are to be blamed.

The teachers also are not to be blamed for the *modus operandi* as we find it, where students, a year away from their schooling, fail in doing the most elementary things in algebra or arithmetic. If I did not feel this, I could not make these remarks at this time. It is because the situation seems to be one in which the college and the high school teachers are equally imposed upon that I take this opportunity to go over the problem.

Lately the Texas Section of this Association has formed a representative committee from High Schools and Colleges which is to survey the situation in mathematics and suggest whatever remedies may be feasible. These suggestions will undoubtedly cover questions of detail, of rearrangement of time and material, of specific improvements that may be possible in the presentation of certain difficult ideas; but working this way, the Committee will surely uncover also some broad difficulties and problems. It is these, rather than the more particular ones, that I should like to discuss here, from my own point of view.

I am going to assume that it is not necessary for me to prove here that there are faults in the products of this education. The State where I find myself may be a state where there are none, and in that case my discourse is out of place. In most regions there is evidently much amiss, and, in particular, the State University of my own state, like that of Wisconsin, has finally acted with considerable severity. At the Rice Institute we have had in the last few years to change fundamentally the method of instruction to take account of the fact that our students come to us essentially unprepared, even in the elements, and unused to any sort of persistent thinking. I speak of these local difficulties because I am familiar with them, but I am fully conscious that the situation is very nearly general.

In the first place, students are made to feel that education is play, not work. The curriculum has to be filled with trivial

excitements in order to induce the pupil to forget temporarily that his main business is football and baseball. There is some chance that the pampered individual may issue forth from school an amateur actor or nature fakir, but little that he shall have any interest in applying his mind. It is my opinion that a pupil should know that he is working for several hours a day, that he is earning his daily bread by faithful effort with his books, and that precisely that is his duty to his parents and to society. He cannot expect to like all his work during his school days, any more than later, but if the work is thorough, if he is required to do it faithfully, if he gradually masters new and systematic ideas, rather than mere tricks of the trade, he will have a steady sense of accomplishment to sustain him. He will enjoy his vacations from school, and he will enjoy—most of him —the school itself.

Of course you have heard this sort of thing before, in terms not more platitudinous. But if these remarks are just, they have a direct application to present practise. If none of you have been asked, more or less diplomatically, to pass trifling students or students low in intelligence, or if there is not a general sentiment on the part of the public that practically all students should be promoted, then I am dealing in platitudes.

I am reminded of an occurrence which took place at the meeting of our State Teachers' Association last Thanksgiving. There a college man had been lamenting the fact that it had been necessary to fail some forty percent of the freshman class in mathematics. But a veteran high school teacher rose to remark that that seemed to him about the right percentage; those were the students who would have been failed in high school if the high schools were permitted to fail students. Needless to say, if teachers could fail the students that deserved it, that number would rapidly diminish.

It is indeed a corollary of these statements that the teachers shall be able to fail students who do not work faithfully, and that those who do not or cannot make a full return on what is invested in them shall go no further. Let them then go outside and apprentice themselves in a trade. They will lead there a dignified and happy life, for they will play an integral part in the life of the community, not floating round like froth on the sea. Give them an opportunity to return to the high school

if their purpose in life changes and their attitude towards education becomes sincere. Provide them, by all means, evening or part time schools by means of which they may themselves test their attitude towards education, and acquire it, less intensively, at any age, while they go on earning money. Let them even pay something for it. Do not pity them, because what they want is responsibility, not pity. Let them grow up!

So much for the trifling student. But in the second place the curriculum is now crowded with subjects which are irrelevant to education, like vocational instruction and commercial training. It must not be. For these things are indeed training, not education. One learns a few methods, and spends much time in practise in order to acquire speed and standard quality. But if that were to be the main element of education, if schools were to fit students for life in that sense, there would have been no cause to institute public schools at all. For at the time when the public schools were inaugurated, there was already adequate training of that kind in the apprentice system. It need never have fallen in abeyance. Something else was looked for, and that something else is what most of our superintendents seem to have forgotten. Anyone who compares a course in typewriting with a course in geometry is likely to notice that difference. Education has relation to life, perhaps mostly to the ability to analyse and understand life; but nothing is so obvious as vocational training.

I should like to make a third criticism which I think is vital. The teachers themselves have not sufficient freedom as individuals. Some of them that I know teach, if it can be called that, given the size of the classes and their state of mind, six periods a day, and spend time besides in tabulating all sorts of data of which the principal value is to enable someone else to acquire a Ph.D. degree in some School of Education. This is what I mean by saying that the teacher is a mere cog in the machine. What teacher who is worth his salt does not resent having his intimate relations with his students curtailed by this machine-like process? The very essence of teaching is an individual and not a statistical relation. For this reason every teacher must distrust the programs that come to him ready made, that prevent him from choosing, in cooperation with his colleagues, the texts that suit his particular needs, or from experimenting with his own ideas.

My last criticism may sound somewhat vague. But you can make it clear by asking yourselves a simple question. Do you regard teaching as a profession or a trade? How does the public regard it? I confess some pessimism as to the probable answer of the public, and even of the school boards at present. But of your answer as teachers I entertain no doubt whatever.

It is here that for a teacher of mathematics the requirement is strictest. For it is in mathematics that the professional preparation is most difficult, and it is for maintenance of this that leisure for study and thought is most essential. Of what avail is it to know the latest news from the schools of pedagogy if one does not understand the difference between commensurable and incommensurable numbers? If one has no time to interest oneself in the historical development of the fundamental ideas about space and time and numbers which have contributed fundamentally to science and philosophy from the time of the Greeks to the present? What use to stand at the gateway of all the measuring sciences if one has not the key? What use to have had the key, and to have lost it or forgotten how to use it?

I doubt if anyone but a teacher of mathematics is competent to guide a new teacher in methods of instruction. But even assuming that there comes something out of the schools of pedagogy that is not obvious and that is not mere instruction in training and drill, how is a teacher to give a student perspective if he does not labor continually at acquiring perspective himself? A teacher undoubtedly measures his success in teaching by his ability to interest his students in the subject which he teaches. But for this he must be profoundly interested himself. He must have had adequate training, and he must have adequate leisure to keep up that interest, by study of the history and content of his subject. If possible let him also write texts, even for merely local use, and mimeograph them if the number of students is too small to warrant the printing of them. These texts would need a warrant of suitability, undoubtedly, especially at the beginning. Nothing would be a better labor of love for this Association than the establishment of committees to advise with such authors, and furnish such warrants.

What is adequate training for a high school teacher of mathematics? In North Carolina I understand you have already a requirement for increased training. A teacher of English

would not regard it as presumptuous to be required to take a course in English each of his four years in college. Why then should a teacher who has trained himself to teach history be required to teach algebra, as many of them are, on the ground that some time or other he must have had a course in algebra? A course during each year of college in mathematics or its applications (and some of these courses in sciences or philosophy must not be neglected) is certainly not too much to ask. Perhaps such teachers will need to be paid higher salaries. But if school boards will reconcile themselves to paying to a man or woman a salary which is adequate to support a family in decency and comfort, candidates for these positions will be forthcoming in the future, as they have been in the past, in spite of the temptations of other careers. If such salaries seem to make schooling too expensive, let our governing boards consider if on the other hand many years of nonsense is worth what it costs, and if they themselves really do not desire, more than all the fads of the present curriculum, some of the great disciplines which have been the heritage of thinking man throughout the ages.

CLEVELAND IN 1929

The next annual meeting of the National Council of Teachers of Mathematics will be held at the Hotel Statler in Cleveland, Ohio, February 22 and 23. The general theme for discussion will be announced in the near future. The theme of the Boston meeting was Mathematics in Modern Life and it was the purpose of the meeting to bring home to the teachers of mathematics and to other educators the increasingly important part played by mathematics in this scientific age. The success of the meeting can be judged by the papers which are now appearing in the Mathematics Teacher.

Suggestions for the next meeting are invited by the President.

Those who want hotel reservations at the Statler will do well to apply for them early. Official blanks for this purpose may be had from Harry C. Barber, President, Exeter, N. H.

REPORT OF COMMITTEE ON INCORPORATION

The committee appointed at the Boston meeting to effect the incorporation of the National Council of Teachers of Mathematics completed its work in June 1928. The By-Laws as approved in Boston will be published in the Register of Members to be issued at an early date.

The Certificate of Organization was issued initially to the three members of this committee as directors. They held the organization meeting, formally adopted the By-Laws and then resigned after turning the organization over to the sixteen directors who had been elected in Boston. This procedure obviated the necessity of calling the whole body of directors to Chicago to attend the organization meeting.

The original Certificate of Organization is to be kept in the archives in the office of the Secretary, but a copy of this certificate and a record of all proceedings in the matter, including full minutes of the organization meeting, are preserved in a fine new loose-leaf record book for the use of the Secretary, who also is provided with an official corporate seal to use in connection with important papers.

By this incorporation act the Council now occupies a dignified and important position among similar organizations, and especially is now qualified to accept gifts and bequests for the furtherance of its purposes as an incorporated body and not for profit, as indicated in the preamble to the By-Laws:

"The object for which the Council is formed is to assist in promoting the interests of mathematics in America, especially in the elementary and secondary fields, by holding meetings for the presentation and discussion of papers, by conducting investigations for the purpose of improving the teaching of mathematics, by the publication of papers, journals, books, and reports, thus to vitalize and coordinate the work of many local organizations of teachers of mathematics and to bring the interests of mathematics to the attention and consideration of the educational world."

The committee wishes to urge upon all members the duty of becoming familiar with all sections of the By-Laws and of as-

siting in every possible way to carry out the purposes of the Council. The most important step which may be taken by every member is to urge all teachers of elementary and secondary mathematics not now members to join the Council. For this purpose the issuance of the Register of Members will be a most important step forward. This will enable all of us to tell at a glance those among our acquaintances who are not members and hence should be brought into the fold. The immediate swelling of our membership to ten thousand will be the most effective form of progress, since this will give us revenue to do many things for the cause, in particular to enlarge and enrich the official journal, in all its departments and to add new departments, to conduct research studies, to finance committee investigations, etc. In this connection it is of the utmost importance that local branches should be organized in every community.

Respectfully submitted,

C. M. Austin,
Lillis Price,
H. E. Slaught, *Chairman*

To the Officers and Directors of the National Council of Teachers of Mathematics

Dear Colleagues:

As chairman of your committee on incorporation, I hereby report that the necessary procedure has been carried out and the Council is now legally incorporated.

Among the items included are: (1) The official Certificate of Organization signed by the Secretary of State of Illinois, in which the original Board of Directors consisted of your committee, C. M. Austin, Lillis Price, and H. E. Slaught; (2) the minutes of the organization meeting held by these three directors, in which the By-Laws were adopted, and in which the Certificate of Organization was amended to change the number of directors from three to sixteen; (3) the resignation of the original three directors in favor of the newly constituted board of sixteen directors; (4) the By-Laws of the incorporated body, being the same as adopted at the Boston meeting except for certain legal phraseology at a few points.

As agreed at Boston the By-Laws will be published in full in

the Register to be issued in the autumn. The Certificate of Organization and a description of the procedure will be published in the October MATHEMATICS TEACHER in the form of a report by your committee to the membership at large. All of the above items are duly recorded in a fine new record book provided for the Secretary together with an official seal to be used by the officers on official documents. These will be forwarded to the Secretary as soon as the documents come from the Recorder of Cook County where all such papers are officially recorded for the Chicago District.

Of course you all understand that the organization in the first instance with a board of three directors was an expedient to avoid calling sixteen directors to Chicago for the organization meetings.

I wish to congratulate the incorporated Council and its sixteen officers and directors upon this important step which I believe is the open door to a great future for the organization. Let us all take hold of things now with a new vigor and determination to "put the Council over" in a large way. In particular the most important step for all of us now is to use our influence in the organization of Branches and the enlargement of our membership.

Yours very sincerely,

H. E. SLAUGHT,

Chairman of the Incorporation Committee.

July, 1928

BUFFALO MATHEMATICS CLUB

Program of Lectures in High School Mathematics for Second Semester 1927–1928

The following program of the Buffalo Mathematics Club for the year 1927–28 shows the interest that is being taken in Buffalo by the Mathematics Teachers with reference to some of the problems facing us.

February 9—Mathematics Textbooks in the Making—Professor Joseph A. Nyberg, M.S., Instructor in Mathematics in Hyde Park High School, Chicago—Bennett.

February 16—Applied Mathematics—Mr. Greenwood—Technical

February 23—Geometric Aids for Elementary Algebra—Miss Alberta Wannenmacher—Hutchinson.

March 1—Continuity of Method in Arithmetic, Algebra, and Geometry—Miss Hallie S. Poole—Lafayette.

March 8—The Mathematics of Chemistry and Physics with Special Emphasis on Formula Work—Mr. McCarthy—South Park.

March 15—Decisive Authorities in Geometric Proofs—Miss Harriet E. Bull—Fosdick Masten.

March 22—Consistent Methods of Developing the Theory and Use of Negative Numbers—Miss Mary Wardwell—Hutchinson.

March 29—Now My Idea Is—Mr. Alfred Smith—Bennett.

April 18—Theory of the Slide Rule—Mr. Greenwood—Technical.

April 25—Intensity and Distribution of Drill, Objective Tests and Lesson Plans—Miss Hallie S. Poole—Lafayette.

May 2—The Humanizing of El. Algebra—Mr. William Betz—Rochester.

May 9—Special Methods in the Teaching of Algebra and Geometry.—Mr. McCarthy—South Park.

The Pythagorean Theorem. ELISHA S. LOOMIS. 1927. Masters and Wardens Association, 22d Masonic District of Ohio, 1110 Webster Ave. S.E., Cleveland, Ohio. 214 pp. Price $2.00.

One hundred sixty-seven geometric proofs and fifty-eight algebraic proofs besides several other kinds of proofs for the Pythagorean Theorem compiled in detailed, authoritative, well-organized form will be a rare "find" for Geometry teachers who are alive to the possibilities of their subject and for mathematics clubs that are looking for interesting material. Dr. Loomis has done a scholarly piece of work in collecting and arranging in such convenient form this great number of proofs of our historic theorem.

The book however is more than a mere cataloguing of proofs, valuable as that may be, but presents an organized suggestion for many more original proofs. The object of the treatise is two-fold, "to present to the future investigator, under one cover, simply and concisely, what is known relative to the Pythagorean proposition, and to set forth certain established facts concerning the proofs and geometric figures pertaining thereto."

There are four kinds of proofs, (1) those based upon linear relations—the algebraic proofs, (2) those based upon comparison of areas—the geometric proofs, (3) those based upon vector operation—the quaternionic proofs, (4) those based upon mass and velocity—the dynamic proofs. Dr. Loomis contends that the number of algebraic and geometric proofs are each limitless, but that no proof by trigonometry, analytics or calculus is possible due to the fact that these subjects are based upon the right triangle proposition.

This book is a treasure chest for any mathematics teacher. The twenty-seven years which Dr. Loomis has played with this theorem as one of his hobbies while he was Head of the Mathematics Department of West High School, Cleveland, Ohio, have been well spent since he has gleaned such treasures from the archives. It is impossible in a short review to do justice to this splendid bit of research work so unselfishly done for the love of mathematics. The book should be highly prized by every mathematics teacher and should find a prominent place in every school, Masonic, and public library.

H. C. CHRISTOFFERSON

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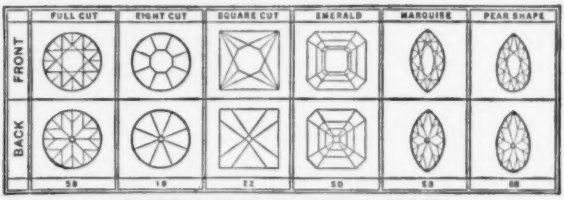

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